Relations for the Calculation of Critical Stress in Pressure Equipment with Cracks

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Analyzing the stress concentration at the crack tip a relationship for deterioration have been obtained taking into account the nonlinear behaviour of the material. On the basis of critical energy principle, in the general case of nonlinear power law behaviour of the structure material, taking into account the deterioration due to crack, there have been established relationships for the critical stress in the case of monotonic loading, as well as in the case of cyclic loading. The comparison with the results reported in literature have shown the generality of the obtained relationships.

Keywords: crack, critical stress, statical loading, cyclic loading, effects superposition

The origin of deterioration in mechanical structures (pressure equipment, gas turbines, ships, aircraft, locomotives, bridges, etc.) is often to be found in the flaws „impressed” during their manufacturing (casting, stamping, forging, welding, riveting, etc.) and / or the cracks „born”, during use, particularly as a result of overloading. Fatigue stress in pressure equipment often causes crack initiation and propagation down to failure.

Some parts of mechanical structures feature micropores which subsequently generate small cracks during loading. In certain circumstances, these cracks propagate. If not detected through periodical examination and repaired, the cracks propagate down to failure. Since pressure equipment problems, from the viewpoint of fracture mechanics, are part of the problem of mechanical structures, we shall further deal with the issue of mechanical structures.

The requirements concerning the safety of mechanical structures need us to consider cracks / flaws, both in their design stage and their monitoring during operation. In some industries, such as the aerospace industry, the method of „damage tolerance design” has replaced the previously used methods, namely the „safe life method” and the „fail safe method”. An example of calculation based on the „fail safe method” is the use of the fracture mechanics based concept “leak before break”, in the design of pressure vessels and piping. They retain their strength of the specimen without cracks; if it satisfies the condition, a ≥ a₀ (1)

where a₀ is the crack dimension at which the crack begins to propagate.

At present, the initial crack length inducing its propagation is taken to be about 0.25 mm for smooth specimens and 0.25 - 0.5 mm for specimens with stress concentrators [1].

a) The value of a₀ can be calculated, in the first approximation:
- under static loading, by using the expression of the stress intensity factor, K, written for its threshold value, K₀.

From \( K = Y \cdot \sigma \cdot \sqrt{\pi \cdot a} \), with \( K = K_{th} \), one obtained,

\[ a_0(\sigma) = \frac{1}{\pi} \left( \frac{K_{th}}{Y \cdot \sigma} \right)^2 \]

where Y is a factor depending on the geometry of the element under load and the crack shape, while \( \sigma \) - fracture strength of the specimen without cracks;

- in the case of cyclic loading, on the basis of range of the threshold of the stress intensity factor, \( \Delta K_{th} \),

\[ a_0(\Delta \sigma) = \frac{1}{\pi} \left( \frac{\Delta K_{th}}{\Delta \sigma_0} \right)^2 \]

where \( \Delta \sigma_0 \) is the range of fatigue limit [2; 3] and \( \Delta \sigma = \sigma_{max} - \sigma_{min} \) is the normal stress range; \( \sigma_{max} \), \( \sigma_{min} \) is the maximum and the minimum stress respectively.

b) The rate of crack propagation until the critical value has been reached may be calculated with a law proposed in the literature [4-6], the most widely used being the law of Paris - Erdogan [4],

\[ \frac{da}{dN} = C \cdot \Delta K_{th}^m \]

where N is the number of stress cycles, C and m are the material constants, while \( \Delta K_{th} = Y \cdot \Delta \sigma \cdot \sqrt{\pi \cdot a} \) is the range of stress intensity factor, \( \Delta \sigma \) is the normal stress range. Several other relations have been proposed for \( (da/dN) \) [5], out of which the most frequently used have been the

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Donahue relationship, the Forman relationship and the Walker relationship [6].

In the case of a pressure vessel, the number of fatigue loading cycles required to grow a crack from the initial half-crack size, \( a_0 \), to the critical half-crack size, \( a_{cr} \), can be determined by integrating relation (2)

\[
N = \frac{2(a_{cr}^{3-m/2} - a_0^{3-m/2})}{C(2-m)\Delta \sigma_a \sqrt{\pi}},
\]

where \( \Delta \sigma_a = \frac{\Delta p \cdot R}{\delta} \) is the range of hoop stress; \( \Delta p = p_{max} - p_{min} \) - the pressure range; \( R \) – the cylinder radius; \( \delta \) – the wall thickness.

The influence of crack on the critical stress, from literature

The hoop stress, \( \sigma_h \), is the highest stress in the wall of a pressurized cylinder,

\[
\sigma_h = p \cdot R/\delta,
\]

where \( p \) is the inner pressure; \( R \) is the cylinder radius; \( \delta \) is the wall thickness.

In the case of corrosion flaws, for example, the remaining hoop stress of a steel pipeline with a volumetric flaw, \( \sigma_h(a) \), in almost all standards is calculated with the Kiefner et al. relation [7, 8],

\[
\sigma_h(a) = \sigma_y \left[ \frac{1-a/\delta}{1-a/(\delta \cdot F)} \right],
\]

\[
F = \left[ 1 + 1.255 \cdot f^2 (R/\delta) - 0.0135 \cdot f^4 (R^2/\delta^2) \right]^{0.5}
\]

\[
\sigma_{cr} = \sigma_y + 68.95 \text{ MPa},
\]

where \( a \) is the defect depth; \( l \) is the defect half-length; \( \sigma_y \) is the tensile yield stress.

The bracketed term can be considered a strength reduction factor.

From Equations (4) and (5) results \( \sigma_h(a) < \sigma_y \), such as the burst pressure of the pipe with a flaw is less the burst pressure of the original pipe, without any defect.

Hasan et al. [8] analysed the burst pressure of internally corroded pipeline with defects, by different codes and standards. Taking into account the Netto et al. results [9] one can write

\[
\sigma_h(a) = 1.1 \cdot \sigma_y \left[ 1 - 0.9435(a/\delta)^6 \cdot (l/2R)^{4.4} \right].
\]

Despite of research effort on determining the remaining strength of corroded pipes with defects, a complete knowledge has not been attained till now [10].

Experimental results for bending and torsion of a rolled bar of annealed 0.46% carbon steel (10 mm in diameter) shows a decrease of the fatigue limit with the increase of the initial flaw size \( (a_i = 1.1 \ldots 1000 \mu m) [11] \).

On the other hand, fracture toughness, \( K_{ic} \), an important criterion to select a material, is used to indicate the resistance to crack growth. But the existing voids accelerate the development of crack, such as \( K_{ic} \) decreases with increase of true strain in the process of plastic deformation [12].

Due to practical importance of defects evolution, acoustic wave propagation in austenitic steel has been used, for the monitoring of the damage at low and high temperature (450°C) [13].

The limiting or critical stress range for fatigue failure \( (\Delta \sigma_{a,cr} = (\sigma_{max} - \sigma_{min})_a) \) decreases with crack size \( (a) \) increasing as it was shown by Chattopadhyay [14].

Generally speaking the fatigue limit decreases sharply for the long cracks (fig. 1) as it was shown by Atzori and Lazzarin [15].

Engineering structures often contains notches. Notches induce stress concentrations and plastic deformation at notches root. Under cyclic loading the fatigue life of a notched structure can be drastically reduced in comparison to constantly applied static load.

For very blunt notch, where \( a \) is high, the fatigue critical stress is \( \sigma_{cr}/K_i \), where \( \sigma_{cr} \) is the smooth sample fatigue limit and \( K_i \) is the elastic stress concentration factor. Below some critical value \( a = a^* \) the notch becomes crack, where \( a^* = k_i \cdot a_0 \).

\[
a_q = \frac{1}{\pi} \left( \frac{K_{i,b}}{\sigma_{cr}} \right)^2.
\]

\( a_0 \) is the standard definition in the Kitagawa diagram for an edge crack,

\[
K_{lf} = 0.7833 \cdot \sigma \cdot \sqrt{\pi} \cdot a \cdot (1 + a/d),
\]

where \( a \) is the crack depth, \( d \) is the diameter of the bar. The bracket term is a stress increasing factor due to the crack. In the case of a notch the applied stress \( \sigma \) is multiplied by \( k_i = 2 \), in order to determine \( \Delta K \) and \( a_{cr} \). This is an appropriate assumption when the crack is very short [17].

A new temperature - damage - dependent fracture strength \( \sigma_{cr}(T,a) \) for ultra-high temperature ceramics was proposed in the following form [19]

\[
\sigma_{cr}(T,a) = \sigma_{cr}(T) \cdot f(T,a).
\]

where

\[
f(T,a) = \frac{\sigma_{cr}(a)}{\sigma_{cr}(T)},
\]

with \( \sigma_{cr}(T) \) - the temperature - dependent fracture strength given at initial damage state; \( T \) - the temperature; \( \sigma_{cr}(a) \) is the damage - dependent fracture strength with respect to reference temperature; \( a \) is the half length of the crack
and \( \sigma_{crr} \) is the fracture strength at the same initial damage state and reference temperature.

It was shown that not only surface cracking but also bulk damage affected the fatigue life. The bulk damage consisted of two factors: internal cracking and local damage.

Since the bulk damage cannot be eliminated by surface removal, surface inspection to detect cracks does not always ensure the structural integrity of damaged components [20].

The design approach refers in the same time to all materials which contain notch, flaws etc., that act brittlely in their operational environment.

A flaw-free steel shows an increasing tensile strength and yield strength with decreasing temperature up to the transition temperature. Commercial materials do have flaws or inherent very little defects to which may be attributed the fracture stress decrease, especially in the material transition temperature from ductile behaviour to brittle behaviour [21].

The cracked steel behaviour analysis at temperatures below nil ductility temperature (NDT) made by Pellini and Puzak [21 - 23], shows the decrease of the ultimate stress with the crack length increasing.

For example, for \( 2a = 200 \text{mm} \), the ultimate normal stress \( \sigma_c(200) = 0.75 \cdot \sigma_c \), and for \( 2a = 300 \text{mm} \) it was experimentally obtained \( \sigma_c(300) = 0.5 \cdot \sigma_c \).

The latter two problems (c and d), will be developed further down.

**Critical stresses for structures with cracks statically loaded**

An elongated crack can be regarded as an ellipse whose longer semi-axis is equal to \( a \) and whose shorter semi-axis is equal to \( b \). If the crack occurs in a plate uniformly loaded with stress fig. 2), the maximum stress occurs at the crack tip and has the expression,

\[
\sigma_{\text{max}} = \sigma + \Delta \sigma , \tag{11}
\]

where the effect of stress concentration is represented by the increase of the stress [24],

\[
\Delta \sigma = 2\sigma \sqrt{a^2 + \rho^2} , \tag{12}
\]

where \( \rho = b^2 / a \) is the curvature radius at the “tip” of the crack.

In a one crack structure, the stress effect is actually superposed on the effect created by the presence of cracks. To solve this problem of effect superposition, we may resort to the concept of energy.

The specific energy (in J/m³) at the crack tip is considered equal to the sum of the specific energy corresponding to stress \( \sigma \) and the specific energy corresponding to an increase \( \Delta \sigma \).

On the basis of the critical energy principle [25, 26] one writes that the total participation of specific energy, \( P_{\sigma} \), is equal to the sum of the specific energy participations corresponding to stress \( \sigma \) and stress increase \( \Delta \sigma \),

\[
P_{\sigma} = P(\sigma) + P(\Delta \sigma) \tag{13}
\]

One considers the general case of the nonlinear behaviour of the material structure, under normal, \( \sigma \), and shear, \( \tau \), stresses given by the following power laws:

\[
\begin{align*}
\sigma &= M_\sigma \cdot \epsilon^\sigma; \\
\tau &= M_\tau \cdot \gamma^\tau,
\end{align*} \tag{14}
\]

where \( \epsilon \) is strain, \( \gamma \) - shear strain, and \( M_\sigma, M_\tau, k \) and \( k_1 \) - constants of the material.

For the behaviour provided by the first relationship (14), \( P(\sigma) \) and \( P(\Delta \sigma) \) have the following expressions [25],

\[
P(\sigma) =\left(\frac{\sigma}{\sigma_{crr}}\right)^{a+1}, \tag{15}
\]

\[
P(\Delta \sigma) =\left(\frac{\Delta \sigma}{\sigma_{crr}}\right)^{a+1}, \tag{16}
\]

where one wrote \( \alpha = 1 / k \), while \( \sigma_c \) and \( \Delta \sigma_c \) are the critical values of \( \sigma \) and \( \Delta \sigma \), respectively. When under just one stress loading, either \( \sigma_{crr} \) or \( \Delta \sigma_{crr} \), the state considered critical is reached (ie, rupture).

Deterioration is written as \( D \), a positive variable taking values between zero and one: \( D = 0 \) - for undamaged material and \( D = 1 \) - for damaged material (rupture, excessive deformation ...).

One obtained the expression for crack initiated deterioration by resorting to the findings in [25],

\[
D(a) = \left(\frac{a}{a_{crr}}\right)^{\frac{a+1}{2}} \tag{17}
\]

where \( P_{\sigma}(0) \) is the critical specific energy participation for an undeteriorated sample. In general, because of the statistical distribution of the mechanical characteristics of the material structure, \( P_{\sigma} \) also features a statistical distribution, that is \( P_{\sigma}(\tilde{\sigma}) \in [P_{\sigma \min}(0); P_{\sigma \max}(0)] \), a distribution where \( P_{\sigma \max}(0) \leq 1 \).

From equations (13), (14), (17) and (18) we obtain,

\[
\frac{\sigma}{\sigma_{crr}} = [P_{\sigma}(0) - D(a)]^{1/(a+1)} \tag{19}
\]

where \( D(a) \) is the critical stress of the structure with one crack, written as,

\[
\sigma_{crr}(a) = \sigma_{crr} \cdot [P_{\sigma}(0) - D(a)]^{1/(a+1)}, \tag{20}
\]

where \( \sigma_{crr} \) is the critical normal stress of the flawless structure material. If the critical characteristics of the...
material parameters are deterministic variables \( P_{cr}(0) = 1 \), so that
\[
\sigma_{cr}(a) = \sigma_{cr}\left[1-D(a)\right]^{\frac{1}{1+\delta}}, \quad (21)
\]
If it is imperative that the material should not exceed the yield stress \( \sigma_{y} \), then \( \sigma_{cr} = \sigma_{y} \) and \( \alpha = 1 \). But in case one allows for the yield stress to be exceeded, then \( \sigma_{cr} = \sigma_{u} \) (ultimate stress). If \( \alpha = 0 \) then \( \sigma_{cr}(a) = \sigma_{cr} \).

Likewise one can determine the critical shear stress of a one crack structure, under load according to fracture modes II or III:
\[
\tau_{j,cr}(a) = \tau_{j,cr}\left[1-D(a)\right]^{\frac{1}{1+\delta}}, \quad (22)
\]
where one wrote \( \alpha = 1 / k_{j} \), while \( j = II \) or III (fracture modes). In relation (22) \( D(a) \) is calculated by using the size of the crack characteristic of the corresponding loading modes (II or III).

If the mechanical characteristics are deterministic variables, \( P_{cr}(0) = 1 \), such as,
\[
\tau_{j,cr}(a) = \tau_{j,cr}\left[1-D(a)\right]^{\frac{1}{1+\delta}}, \quad (23)
\]
In the case of simultaneous loading with normal stress \( \sigma \) and shear stress \( \tau \), the result is the superposition of effects. Let us consider a bar with radius \( R \) and a circular crack of radius \( a \) inside it, loaded by tensile force \( F \) which determines normal stress \( \sigma \), and torque \( M \) which causes the shear stress \( \tau \) inside the crack (fig. 3, a).

According to the principle of critical energy [25] the total participation of the specific energies corresponding to \( \sigma \), \( \tau \), and \( a \):
\[
P_{\gamma} = P(\sigma) + P(\tau) + P(a),
\]
wherein one replaces \( P(\tau) = (\tau / \tau_{cr})^{\alpha+1} \) and considers relations (15), (17) and (18) thus obtaining,
\[
\left(\frac{\sigma}{\sigma_{cr}}\right)^{\alpha+1} + \left(\frac{\tau}{\tau_{cr}}\right)^{\alpha+1} = P_{cr}(0) - D(a), \quad (24)
\]
where \( D(a) \) is calculated with \( a = a(\sigma) \) and with \( a = a(\tau) \) and the maximum resulting value is considered. If \( a = a(\sigma) \) then \( D(a) \) is calculated with exponent \( \alpha \), while if \( a = a(\tau) \) then one calculates by using exponent \( \alpha \). Relation (24) represents a curve that is the geometrical locus of the loading points with \( \sigma / \sigma_{cr} \) and \( \tau / \tau_{cr} \) when rupture occurs. If \( P_{cr}(0) = \left\{ P_{cr,\text{min}}(0); P_{cr,\text{max}}(0)\right\} \), then there are obtained two curves, wherein the experiment points are introduced (fig. 3, b).

Critical stresses for structures with cracks, cyclically loaded

Under cyclic loading with normal stresses ranging between \( \sigma_{\text{min}} \) and \( \sigma_{\text{max}} \), stress amplitude is
\[
\sigma = 0.5(\sigma_{\text{max}} - \sigma_{\text{min}}),
\]
and mean stress is
\[
\sigma_{m} = 0.5(\sigma_{\text{max}} + \sigma_{\text{min}}),
\]
The total participation of the specific energies [25],
\[
\delta_{cr}(\sigma) = P(\sigma_{a}) + P(\sigma_{m}) + P(a), \quad (25)
\]
where \( P(\sigma_{a}) \) and \( P(\sigma_{m}) \) are calculated with relationships like in (15), while \( P(a) = D(a) \) with relationship (17). One obtains,
\[
\left(\frac{\sigma_{a}}{\sigma_{m,cr}}\right)^{\alpha+1} + \left(\frac{\sigma_{m}}{\sigma_{a,cr}}\right)^{\alpha+1} \delta_{m} = P_{cr}(0) - D(a), \quad (26)
\]
where \( \sigma_{a,cr} = \sigma_{a}(N) \) is the fatigue strength of the structure fully reversed stress loaded under \( N \) cycles, while \( \sigma_{m,cr} = \sigma_{m}(\text{yield stress}) \) or \( \sigma_{m}(\text{ultimate stress}) \), depending on whether fracture occurs when \( \sigma_{y} < \sigma < \sigma_{u} \) or when \( \sigma_{y} > \sigma_{m} \) (traction) and \( \delta_{m} = 1 \) if \( \sigma_{m} > 0 \) (compression). From relation (26), with \( \sigma_{a,cr} = \sigma_{a}(N) \) and \( \sigma_{m,cr} = \sigma_{m}(\alpha;N) \), one obtains the stress amplitudine upon reaching the critical state for the cracked structure after \( N \) loading cycles,
\[
\sigma_{a,cr}(\alpha;N) = \sigma_{a,cr}(N)\left[P_{cr}(0)\right]^{\frac{1}{1+\delta}} \delta_{m}^{-\frac{1}{1+\delta}}, \quad (27)
\]
where one wrote \( \sigma_{a,cr}(N) \) – strength of crackless structure, under fully reversed normal stresses, after \( N \) cycles; \( \sigma_{m,cr}(\alpha;N) \) – strength of cracked structure \( (\alpha=0) \) under fully reversed normal stresses, after \( N \) cycles.

The value of \( \sigma_{m,cr}(\alpha;N) \) is influenced both by the value of the mean stress, \( \sigma_{m} \), and by the crack deterioration, \( D(a) \). The mean traction stress \( (\delta_{m} = 1) \) reduces, while the mean compression stress \( (\delta_{m} = -1) \) enhances \( \sigma_{m,cr}(\alpha;N) \). In the case of fully reversed normal stress \( \sigma_{m} = 0 \), equation (27) becomes,
\[
\sigma_{a,cr}(\alpha;N) = \sigma_{a,cr}(N)\left[P_{cr}(0)\right]^{\frac{1}{1+\delta}}, \quad (28)
\]
When the critical characteristics of the material parameters are deterministic variable \( P_{cr} = 1 \), the fracture stress after \( N \) cycles loading is (fig. 4),
\[
\sigma_{a,cr}(\alpha;N) = \sigma_{a,cr}(N)\left[1-D(a)\right]^{\frac{1}{1+\delta}}, \quad (29)
\]
where \( \sigma_{a,cr}(N) \) is the strength of the crackless specimen under fully reversed normal stress, after \( N \) cycles.
The case of fatigue under successive blocks with different normal stresses amplitude is presented in [25; 28].

Under cyclic fully reversed shear stresses one obtains a relation analogous to (27):

\[ \tau_{-1,s}(N) = \tau_{-1,s}^{0}(N) \left[ P_{\sigma}(0) - D(a) \right]^{1/n_{a+1}} \]

(30)

where we noted: \( \tau_{-1,s}(N) \) - shear strength of structure without cracks under fully reversed shear stresses after \( N \) cycles; \( \tau_{-1,s}^{0}(N) \) - shear strength of structure with cracks (\( a \neq 0 \)) under fully reversed shear stresses after \( N \) cycles. In the case of an fully reserved shear stress (\( \tau_{m} = 0 \)), equation (30) becomes,

\[ \tau_{-1,s}(a;N) = \tau_{-1,s}^{0}(N) \left[ P_{\sigma}(0) - D(a) \right]^{1/n_{a+1}} \]

(31)

If the critical characteristics of the material parameters are deterministic variable \( P_{cr} = 1 \).

For a cracked specimen the fracture shear stress (with \( P_{cr}(0) = 1 \)) results from equation (31),

\[ \tau_{-1}(a;N) = \tau_{-1}(N) \left[ 1 - D(a) \right]^{1/n_{a+1}} \]

(32)

where \( \tau_{-1}(N) \) is the fully reversed shear strength of the crackless specimen, after \( N \) cycles.

When a cyclic loading with normal stress is superposed on the cyclic loading with shear stress, in the same phase, the total participation of the specific energy equals the sum of total participations corresponding to normal stress \( P_{f}(\sigma) \) and shear stress \( P_{f}(\tau) \),

\[ P_{f} = P_{f}(\sigma) + P_{f}(\tau) \]

(33)

where \( P_{f}(\sigma) \) is given by equation (25), while \( P_{f}(\tau) \), by an analogous relationship. From equations (18) and (33) there results,

\[ \left( \frac{\sigma_{a}}{\sigma_{-1,s}^{0}(N)} \right)^{n_{a+1}} + \left( \frac{\tau_{a}}{\tau_{-1,s}^{0}(N)} \right)^{n_{a+1}} = B \]

(34)

where one noted,

\[ B = P_{f}(0) - \left( \frac{\sigma_{a}}{\sigma_{-1,s}^{0}(N)} \right)^{n_{a+1}} \cdot \delta_{a} - \left( \frac{\tau_{a}}{\tau_{-1,s}^{0}(N)} \right)^{n_{a+1}} \cdot \delta_{a} - D(a) \]

where \( D(a) \) is calculated as shown under relation (24).

Relation (34) describes a curve that represents the geometrical locus of the cyclic loading points in the diagramme \( \sigma_{a}/\sigma_{-1,s}(N) - \tau_{a}/\tau_{-1,s}(N) \) where fracture occurs (fig. 5).

In case the mechanical characteristics are deterministic quantities, \( P_{\sigma}(0) = 1 \).

**Comparisons**

Based on the concept introduced by Kacianov and Rabotnov [29, 30] the effective stress at the crack tip, \( \sigma_{eff} \) is related to deterioration and the applied stress, \( \sigma \), with the relationship,

\[ \sigma_{eff} = \frac{\sigma}{1 - D} \]

(35)

Xue and Wierzbicki [31] proposed a relation of the form,

\[ \sigma_{eff} = \frac{\sigma}{1 - D^{\beta}} \]

(36)

where \( \beta \) is a constant of the material that is to be obtained by comparing the relationship with the experimental data. If the material has a linear behaviour \( \beta = 1 \) [31].

Similarly, the effective stress variation is correlated with the variation in the applied stress \( \Delta \sigma \),

\[ \Delta \sigma_{eff} = \frac{\Delta \sigma}{1 - D} \]

(37)

These relationships are particular cases of relationship (21) where \( \sigma_{cr} \) plays the part of \( \sigma_{eff} \) (or \( \Delta \sigma_{eff} \)), and \( \sigma_{cr}(a) \) corresponds to \( \sigma \) and \( \Delta \sigma \).

Deterioration has \( D(a) \) been defined in literature as the ratio [32],

\[ D(a) = \frac{a}{a_{f}} \]

(38)

between the current or instantaneous crack length \( a \), and its final length, \( a_{f} \). Thus defined, deterioration does not take into consideration the behaviour of the material that in relationship (17), proposed in this paper, is considered through exponent \( \alpha \). In the case of linear - elastic behaviour \( \alpha = 1 \) equation (17) becomes equation (38).

The fracture of pressure shell type components under normal operating conditions, at stressses that are lower than those allowable, has been attributed to cracks, a finding that stimulates the use of fracture criteria based on the concept of deterioration established in this paper.

**Conclusions**

There were established - on the basis of critical energy principle - mechanical failure criteria for structures with nonlinear behaviour with cracks, statically loaded: - with normal stress (20) and (21); - with a shear stress (22) and (23), - simultaneously, with a normal stress and with a shear stress (24) and cyclically loaded, respectively: - with a normal stress block (27) - (29), - with a shear stress (fig. 5).
block (30) - (32), simultaneously with a block of normal stresses and one of shear stresses (34).

As in these relationships there occurs the concept of deterioration, a general relation has been developed for calculating the deterioration caused by the presence of a crack (17), which takes into account the non-linear behaviour of the material structure.

The general theoretical results obtained in this study have been compared with some particular results reported in the literature which fail to take into consideration the behaviour of the material structure.

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Manuscript received: 22.05.2013