
II. Energy, work, heat and efficiency in viscous and viscoelastic liquids and in solids**

VALERIU V. JINESCU1, STOIAN PETRESCU 2, COSMIN JINESCU1*
1 Politehnica University of Bucharest, Department of Process Equipments, 313 Splaiul Independentei, Bucharest, Romania
2 Politehnica University of Bucharest, Department of Engineering Thermodynamics, 313 Splaiul Independentei, Bucharest, Romania

Abstract. We present a unitary thermodynamic approach in the treatment of deformation and flow processes in: viscous and elastic liquids and in solids. The formulae for work and heat interactions (exchanged in these processes) are deduced, and a comparison between them is made, in order to illustrate the similarities and differences (particularities): – for purely viscous liquids (with linear and nonlinear behaviour); – for viscoelastic liquids in flowing; – for solids (with linear and nonlinear behaviour), in the case of using engineering concepts or natural concepts (for mechanical stress and strain). Electrical conductors (made from solid materials) are studied also from the point of view of work and heat interactions. The influence of the concepts used in the cases of matter deformation or fluid flow on the formulae of work and heat is studied. The need to express correlations between causes (stress) and effects (strain, shear strain or strain rate), depending on the way the effects have been determined (calculations or measurements) is emphasized. The numerical examples show how the results obtained in the paper may be practically used.

Keywords: energy; work and heat; electromagnetic processes; viscous liquids; viscoelastic liquids; solids; matter behaviour; efficiency.

Work and heat in viscous and viscoelastic liquid flow processes

In §.4 we referred to the volume deformation (compression and expansion) of perfect gases. We shall further refer to the flow of purely viscous and viscoelastic liquids, an important issue for energy efficient processes [27].

Concepts and rheological behaviour of viscous and viscoelastic liquids

At present, in order to describe the rheological behavior of a flowing viscous fluid one uses the concepts of shear stress τ and shear rate or velocity gradient γ = dγ / dt (fig. 7).

Fluids can behave as purely viscous (linearly or nonlinearly) and viscoelastic behavior (linearly or nonlinearly) time dependent [4; 28].

The behaviour of a linear-viscous fluid also called Newtonian is described by Newton's law, where the shear stress τ depends on the pressure exerted upon the fluid, while the shear rate γ depends on the fluid velocity. At a certain point in the fluid (fig. 7),

\[ \dot{\gamma} = \frac{d\gamma}{dr} = \frac{d}{dr}\left(\frac{dx}{dr}\right) = \frac{dv}{dr}. \]

The behaviour of a nonlinear (nonnewtonian) fluid can be described by the Oswald - de Waele's power law (τ = K_γ γ^n – table 1).

In general, one considers that the variation of the fluid pressure along the nozzle is linear and the apparent pressure gradient is calculated with relation (fig. 8):

\[ \frac{dp}{dx} = \frac{p_1 - p_2}{l}. \] (57)

![Fig. 7. Viscous fluid passing through a tube of the radius R, under the action of pressure difference Δp = p_1 - p_2.](image)

![Fig. 8. Pressure variation in a purely viscous fluid along a nozzle: linear (BC) and nonlinear (AB).](image)
In reality, close to the entrance zone (A in fig. 12) the pressure variation is nonlinear (AB) after which, along path BC the pressure variation is linear. The use of a linear variation facilitates actual calculations. For this reason, one extends the linear variation up to the intersection of point D of the horizontal passing through the ordinate point \( p_1 \) (pressure at inflow section). Slope straight-line CBD (58) represents the real pressure gradient. Length \( l_c \) is the correction through which the nonlinear variation AB is transformed into the pressure linear variation.

Relation (58) can also be written as:

\[
\left( \frac{dp}{dx} \right)_v = \frac{p_1 - p_0}{l + l_c},
\]

where \( \Delta p \) is the pressure difference corresponding to the volume deformation energy, which corresponds to the "energy consumed" in the inlet zone, which is in fact the energy for overcoming the external friction. Seemingly, the nozzle length is \( (l + l_c) \).

For a purely viscous fluid \( p_0 = p_1 \) - the pressure of the environment where the fluid enters on leaving section C.

In the case of a viscoelastic fluid \( p_2 > p_0 \) (fig. 9). The elastic stresses accumulated in the liquid along the ABC relaxes after leaving the nozzle beyond curve CC1 over the relaxation length \( l_{rel} \). Consequently, the liquid vein of diameter \( 2R \), in leaving over distance \( l_{in} \) swells up and becomes \( 2R_e > 2R \).

The real variation of the viscoelastic fluid pressure is calculated with relation (58) where, however \( p_2 > p_0 \) namely,

\[
p_2 = p_0 + \Delta p_{el},
\]

where \( \Delta p_{el} \) is the internal pressure caused by the elastic stresses, in the outlet section C. These stresses expand over distance \( l_{rel} \).

Fig. 9. Pressure variation in a viscoelastic fluid along a nozzle

Relation (56) for viscoelastic fluid can be also written as:

\[
\left( \frac{dp}{dx} \right)_{el} = \frac{p_1 - p_0}{1 + l_c + l_{rel}},
\]

or

\[
\left( \frac{dp}{dx} \right)_{el} = \frac{p_1 - p_0 - \Delta p - \Delta p_{el}}{l},
\]

In the case of relation (61), it is necessary to know the additional lengths \( l_c \) and \( l_{rel} \) while in the case of relation (62) the pressure drops \( \Delta p \) and \( \Delta p_{el} \) are necessary to be determined.

The dependence \( \tau(\gamma) \) as currently drawn is hybrid since stress \( \tau = \tau_2 \) is calculated depending on external loading, \( \Delta p = p_1 - p_0 \), while \( \gamma = \gamma(D) \) depends on the measured volume flow rate \( D_0 \).

The use of Bagley's correction in drawing these diagram for polymer melts for examples [29; 30] makes it possible for dependence to be reached \( \tau(\gamma) \) which is a real diagram.

In the case of viscoelastic fluids, a part of the pressure drop, \( \Delta p_{el} \) is stored in the fluid under the form of elastic stresses which relaxes upon leaving the nozzle. Consequently, according to previous relation the pressure gradient is written as

\[
\left( \frac{dp}{dx} \right)_{el} = \frac{\Delta p}{l} - \frac{\Delta p_{el}}{l}. \tag{63}
\]

**Mechanical work**

Work per unit of time or power for the flow of a unit volume of the fluid, at a certain moment, is:

\[
\dot{w} = \tau \cdot \dot{\gamma}. \tag{64}
\]

Work corresponding to the permanent flow (steady state regime) of the fluid, over a time \( t \), is:

\[
\dot{w} = t \left( \dot{w} \cdot \frac{dx}{dt} = \dot{w} \cdot \frac{dx}{\tau} \right). \tag{65}
\]

a. Internal work is calculated by using the viscous fluid real pressure gradient (\( dp / dx \)) or a viscoelastic fluid pressure gradient (\( dp_{el} / dx \)).

For a purely viscous fluid the real shear stress is calculated depending on the real pressure gradient, (\( dp / dx \)). In the case of a nozzle flow (fig. 8), the real shearing stress at the wall is:

\[
\tau_r = \frac{R}{2} \left( \frac{dp}{dx} \right)_r, \tag{66}
\]

while the internal power is:

\[
\dot{w}_{in} = \tau_r \cdot \dot{\gamma}. \tag{67}
\]

Relation (67) features the following practical expressions for internal power,

\[
\dot{w}_{in} = \begin{cases} 
\frac{1}{\mu} \cdot \tau_r & \text{for linear-viscous fluids (}\tau = \mu \cdot \dot{\gamma}); \\
\frac{1}{K_{vis}} \cdot \tau_r & \text{for nonlinear power law-viscous fluids (}\tau = K_{vis} \cdot \dot{\gamma});
\end{cases} \tag{68}
\]

For a viscoelastic fluid, the viscoelastic shear stress is calculated depending on the viscoelastic pressure gradient, (\( dp / dx \)). In flowing through a nozzle (fig. 9), the shearing stress at the wall of the viscoelastic fluid,

\[
\tau_{el} = \frac{R}{2} \left( \frac{dp_{el}}{dx} \right)_{el}, \tag{69}
\]

while the internal power is written as:

\[
\dot{w}_{in,el} = \tau_{el} \cdot \dot{\gamma}, \tag{70}
\]

which has the following practical expressions:
Since \((\frac{dp}{dx})_{\text{vel}} < (\frac{dp}{dx})_{\text{r}}\), the internal work in the viscoelastic fluid is less than in the purely viscous fluid.

b. External work is also calculated with relation (65) where one uses the apparent pressure gradient \((\frac{dp}{dx})_{a}\).

Consequently, the apparent shearing stress at the wall, upon flowing through the nozzle,

\[ \tau_{a} = \frac{R}{2} \left( \frac{dp}{dx} \right)_{a} \tag{72} \]

and the external power is written as:

\[ \dot{w}_{ex} = \tau_{a} \cdot \dot{\gamma} \tag{73} \]

which has the following practical expressions:

\[
\dot{w}_{ex} = \begin{cases} 
\frac{1}{\mu} \tau_{a} - \text{for the linear behaviour of the viscous component} \\
\frac{1}{K^{v}} \tau_{a}^{1+1} - \text{for the nonlinear power law behaviour of the viscous component;}
\end{cases}
\tag{74}
\]

Since \((\frac{dp}{dx})_{a} > (\frac{dp}{dx})_{r} > (\frac{dp}{dx})_{\text{vel}}\) it follows that \(\dot{w}_{ex} > \dot{w}_{in} > \dot{w}_{\text{vel,ref}}\).

By comparing the relations (68), (70) and (74), taking into account the relation (59), (66) and (72), as well as (69) and (71) one results,

\[
\frac{\dot{w}_{ex}}{\dot{w}_{in,ref}} = \begin{cases} 
\left[ (\frac{dp}{dx})_{a}/(dp/dx)_{r} \right]^{\frac{1}{1+1}} - \text{for the linear behaviour} \\
\left[ (\frac{dp}{dx})_{a}/(dp/dx)_{\text{vel}} \right]^{1+1} - \text{for the linear behaviour}
\end{cases}
\tag{75}
\]

Shearing stress at the wall in the viscoelastic fluid is given by relation (69). At the same flow rate (hence the same \(\gamma\)), \(\tau_{\text{vel}}\) is lower than \(\tau_{r}\).

In figure 10 there are represented three curves: \(\tau(\gamma)\), \(\tau_{\text{vel}}(\gamma)\), and \(\tau_{\text{vel}}(\gamma)\) which express the dependence on \(\gamma\) of the: - shearing stress calculated with relation (72) (curve 1); - shearing stress, in a purely viscous fluid, calculated with relation (66) (real curve 2); - shearing stress, in a viscoelastic fluid, with relation (69) (curve 3).

**Heat and elastic effects**

Internal work, corresponding to the internal friction, is transformed into heat, \((\dot{w}_{i} = \dot{q})\), which means also that: \(\dot{w}_{i} = \dot{q} = \dot{q} \cdot \dot{\gamma}\).

In the case when no heat is lost to the environment \((\dot{Q} = 0)\), the internal work will be transformed entirely into internal energy, \(\dot{w}_{i} = \dot{u}\).

For a compressible viscous fluid the difference between external and internal work corresponds to the stress difference (fig. 10)

\[ \Delta \tau_{\text{rel}} = \tau_{r} - \tau_{a} \tag{77} \]

This determines the Power corresponding to the compressibility, \(\dot{w}_{c} = \Delta \tau_{\gamma} \cdot \dot{\gamma}\), or to the volume deformation (it changes only the fluid density), which relaxes until it leaves the nozzle.

For a compressible viscous elastic fluid, the stress difference:

\[ \Delta \tau_{\text{rel}} = \tau_{a} - \tau_{r} \tag{78} \]

contains both: the volume variation work, as well as the elastic deformation work which relaxes when leaving the nozzle. The stress difference corresponding to the elastic effect is:

\[ \Delta \tau_{\text{el}} = \Delta \tau_{a} - \Delta \tau_{r} \tag{79} \]

From the analysis carried out it follows that the power corresponding to compressibility in the fluid flow is:

\[ \dot{w}_{c} = \dot{w}_{ex} - \dot{w}_{in} \tag{81} \]

or

\[ \dot{w}_{c} = \dot{w}_{\text{vel,ref}} - \dot{w}_{in} \tag{82} \]

The power corresponding only to the fluid elastic component, the one that expands upon leaving the nozzle, is:

\[ \dot{w}_{\text{el}} = \Delta \tau_{\text{rel}} \cdot \dot{\gamma} \tag{83} \]

The efficiency in the process of fluid flow,

\[ \eta_{p} = \frac{\dot{w}_{\text{el}}}{\dot{w}_{ex}} \tag{84} \]

**Work and heat interactions in deformation processes of solids**

**Concepts used for work and heat computations**

Two sets of concepts are used, in order to express the behaviour of solid bodies, under the mechanical loading:

- Engineering concepts (fig. 11),

\[ \sigma = \frac{F}{A_{0}} \; \text{strain, } \varepsilon = \frac{\Delta l}{l_{0}} \tag{84} \]

where: \(F\) - applied force; \(A_{0}\) - initial cross section area; \(l\) - initial sample length; \(\Delta l\) - sample length variation; \(l\) - sample length after loading with the force \(F\).
natural concepts, according to Hencky, where: A - area of the cross section after loading with the force F.

The following correlations between the two sets of concepts, exists:

\[
\begin{align*}
\sigma_n &= \sigma \cdot (1 + \varepsilon), \\
\varepsilon_n &= \ln(1 + \varepsilon).
\end{align*}
\] (86)

The characteristic curve \(\sigma - \varepsilon\) differs from the characteristic curve \(\sigma_n - \varepsilon_n\). The last one is usually a continuously increasing curve (fig. 12).

For several low or medium carbon steel the curve \(\sigma - \varepsilon\) (fig. 12, a), may be considered linearly and is represented by Hooke’s law \((\sigma = E \cdot \varepsilon)\) up to the yield stress (on the line OA or \(\sigma \leq \sigma_y\)), while the curve \(\sigma_n - \varepsilon_n\) (fig. 12, b) could be represented by the same relation \((\sigma = M_n \cdot \varepsilon)\) up to the failure.

The behaviour laws and mechanical work

Work per unit volume \(w = W / V\) under normal stress \(\sigma\) is written as:

\[
w_\sigma = \int \sigma \cdot d\varepsilon,
\] (87)

while loading under shear stress is:

\[
w_\tau = \int \tau \cdot d\gamma.
\] (88)

Work may be expressed depending on measured variables \((\varepsilon_1, \varepsilon_2)\) or \((\gamma_1, \gamma_2)\) respectively, or depending on calculated variables \((\sigma_{1(c)}, \sigma_{2(c)})\) or \((\tau_{1(c)}, \tau_{2(c)})\), respectively.

a. The laws of behaviour currently used is a linear correlation between engineering concepts: the Hooke’s law of linear-elastic behaviour, for loading under normal stresses \((\sigma = E \cdot \varepsilon)\) and the similar law \((\tau = G \cdot \gamma)\) for loading under shear stresses.

Depending on measured strains \((\varepsilon_1, \varepsilon_2)\) respectively, in the process of passing the sample from the length \(l_1 > l_0\) to the length \(l_2 > l_1 > l_0\) the internal work per unit volume will be:

\[
w_\sigma(\varepsilon) = \frac{E}{2} (\varepsilon_1^2 - \varepsilon_2^2).
\] (89)

By taking into consideration Hooke’s law one gets:

\[
w_\sigma(\varepsilon) = \sigma_{1(c)} \cdot \varepsilon_1 - \sigma_{2(c)} \cdot \varepsilon_2 = \frac{\sigma_1^2 - \sigma_2^2}{E},
\] (90)

where \(\sigma_1\) and \(\sigma_2\) results on the bases of the measured variables \(\varepsilon_1\) and \(\varepsilon_2\).

In the case of shear stresses, on the basis of measured shear strains \(\gamma_1\) and \(\gamma_2\), the internal work per unit volume, written \(w_\tau = W(\gamma)\), is given by relation (88). After replacing \(\tau = G \cdot \gamma\) one gets the internal work per unit volume of the sample under shear stress loading:

\[
w_\tau(\gamma) = \frac{G}{2} (\gamma_2^2 - \gamma_1^2)
\] or

\[
w_\tau(\gamma) = \frac{\tau_1 \cdot \gamma_2 - \tau_2 \cdot \gamma_1}{2G} = \frac{\tau_1^2 - \tau_2^2}{2G},
\] (91)

where \(\tau_1\) and \(\tau_2\) are calculated based on the measured shear strains, \(\gamma_1\) and \(\gamma_2\).

\[\text{· Starting from the calculated normal stress with the eq. (84), written as } \sigma_{(c)} \text{, the formula for external work per unit volume will be}
\]

\[
w_\sigma(\sigma_{(c)}) = \frac{(\sigma_{2(c)})^2 - (\sigma_{1(c)})^2}{2E},
\] (92)

where \(\sigma_{1(c)}\) and \(\sigma_{2(c)}\) is the stress calculated corresponding to force \(F_1\) and \(F_2 > F_1\), respectively.

This is the work of external forces, since the stress \(\sigma_{(c)}\) is calculated as function of the external force \(F\).

Analogously, it follows that the external forces per unit volume under loading with calculated shear stresses, \(\tau_{1(c)}\) and \(\tau_{2(c)}\) will be:

\[
w_\tau(\tau_{(c)}) = \frac{\tau_{2(c)}^2 - \tau_{1(c)}^2}{2G},
\] (93)

b. The evaluation of materials behaviour, described with the natural concepts, is carried out by using the nonlinear law functions, written as:
\[ \sigma_n = M \cdot e_n^k, \quad \text{(94)} \]

in the case of normal stress loading, where \( M \) and \( k \) are material constants, or with

\[ \tau_n = M \cdot \gamma^k, \quad \text{(95)} \]

where \( M \) and \( k \) are also material constants. (The exponent \( k \) and \( k_1 \) refers here to the solid under analysis and does not relate to the adiabatic exponent used for perfect gases!). In these cases, the elasticity modules are locally defined by relations:

\[ E(\sigma_n) = \frac{\sigma_n}{\varepsilon_n} = M \cdot k \cdot e_n^{k-1} \text{ and } G(\tau_n) = \frac{\tau_n}{\gamma} = M \cdot k_1 \cdot \gamma^{k-1}. \]

These modules of elasticity are not constant as in the case of linear-elastic behavior, but depend on \( \varepsilon_n \) and on \( \gamma \), respectively.

Depending on the natural strain (resulting from measurements), the internal work per unit volume is given by relation (87) where \( \sigma \) is replaced by \( \sigma_n \) and \( \varepsilon \) by \( e_n \). One obtains

\[ w_n(\varepsilon_n) = \int E(\sigma_n) \, \varepsilon_n \, d\varepsilon_n, \]

which represents the area under the characteristic curve \( \sigma_n - \varepsilon_n \) (fig. 13).

After integration one gets:

\[ w_n(\varepsilon_n) = \left[ M \cdot e_n^{k+1} - e_n^{k+1} \right] / (k+1), \quad \text{(96)} \]

or

\[ w_n(\varepsilon_n) = \left[ \sigma_n \cdot e_n - \sigma_n \cdot \varepsilon_n \right] / (k+1). \quad \text{(97)} \]

![Fig. 13. Work per unit volume, in the process of deformation from \( \varepsilon_{n,1} \) to \( \varepsilon_{n,2} \) (measured strains).](image)

Analogously, on the basis of shear strain measurement one gets the internal work under shear stress loading:

\[ w_n(\gamma) = M \cdot \left[ \gamma_n^{k+1} - \gamma_n^{k+1} \right] / (k+1), \quad \text{(98)} \]

or

\[ w_n(\gamma) = \left[ \tau_n \cdot \gamma_n - \tau_n \cdot \gamma_1 \right] / (k+1). \]

With \( k = k_1 = 1 \) from relations (97) and (98) one obtains for the internal mechanical work relations, as function of the measured variables, for the linear-elastic behavior the relations (89), (90) and (91).

One finds that \( w_n(\varepsilon_n) \) and \( w_n(\gamma) \) in the case of the nonlinear behavior, depend on the exponents \( k \) and \( k_1 \), from the behaviour laws, in a similar way as in the case of polytrophic process for the perfect gases.

Depending on the calculated natural stress \( \sigma_n(\varepsilon_n) \) and \( \sigma_n(\gamma_n) \), one gets the relation for external mechanical work per unit volume:

\[ w_n(\varepsilon_n) = \frac{1}{(k+1)} \cdot \frac{\varepsilon_n}{k} \left[ \frac{1}{\sigma_n^{k+1}} - \frac{1}{\sigma_n^{k+1}} \right], \quad \text{(99)} \]

or

\[ w_n(\varepsilon_n) = \left[ \sigma_n(\varepsilon_n) \cdot \varepsilon_n - \sigma_n(\varepsilon_n) \cdot \varepsilon_n \right] / (k+1). \quad \text{(100)} \]

where \( \sigma_n(\varepsilon_n) \) and \( \sigma_n(\gamma_n) \) are normal stress values calculated depending on the applied forces \( F_1 \) and \( F_2 > F_1 \) with the relation (85), while \( \varepsilon_n(\varepsilon_n) \) and \( \varepsilon_n(\gamma_n) \) are the natural strains calculated with relation (94). One may write \( \varepsilon_n(\sigma_n(\varepsilon_n)) = \varepsilon_n(\sigma_n(1)), \varepsilon_n(\sigma_n(\gamma_n)) = \sigma_n(\gamma_n), \sigma_n(\varepsilon_n) = \sigma_n(\varepsilon_n), \sigma_n(\gamma_n) = \sigma_n(\gamma_n), \)

\[ w_n(\sigma_n) = \left[ \sigma_n(\varepsilon_n) \cdot \varepsilon_n - \sigma_n(\varepsilon_n) \cdot \varepsilon_n \right] / (k+1), \quad \text{(100)} \]

where \( (\gamma_n^1) \) and \( (\gamma_n^2) \) are calculated based on \( \tau_n(\varepsilon_n) \) and \( \tau_n(\gamma_n) \), respectively, with relation (95).

One finds that the external work per unit volume expressed as a function of the calculated variables \( (\sigma_n(\varepsilon_n) \) and \( \tau_n(\gamma_n) \), in the case of the nonlinear behaviour, depends on exponent \( k \) and \( k_1 \) (from behavior laws). For \( k = k_1 \) the formula for work \( w \) become those from the case of linear behaviour, (92) and (93).

**Behaviour of materials and resulting heat**

**a.** At present the diagram representing sample behaviour under tensile stress are hybrid diagram, since they use the normal stress calculated with the first relation (84), based on the force applied to the sample, while the strain results from measuring the increase in the sample temperature from \( T_o \) to \( T \), can be expressed as a function of the increase in the heat \( q \), which represents the area under the characteristic curve \( \sigma_n - \varepsilon_n \) (fig. 13). As the energy consumed for overcoming the reluctance of the material and which changes into heat \( (q) \) is neglected, dependence \( \sigma_n - \varepsilon \) does not correctly represent the material behaviour, since the work resulting from measurements is different from the work obtained on the basis of the calculated stress \( w(\varepsilon) \neq w(\sigma) \)

**b.** Generally the internal work \( w_n(\varepsilon) \) represents the net effect of loading while external work \( w(\sigma) \) expresses the effect of external loading which contains also the heat \( Q \) or the amount of heat per unit volume \( q = Q / V \). In the case of linear - elastic behavior,

\[ w_n(\varepsilon) = w_n(\varepsilon) + q, \quad \text{(102)} \]

with from the internal work per unit volume, resulting from measurements, is calculated with relation:

\[ w_n(\varepsilon) = w_n(\varepsilon) - q. \quad \text{(103)} \]

The efficiency of the deformation process of solids,

\[ \eta_n = w_n(\varepsilon) / w_n(\sigma), \quad \text{(104)} \]

**c.** In the case of non-linear behaviour, relation (103) becomes,

\[ w_n(\varepsilon) = w_n(\varepsilon) - q. \quad \text{(105)} \]

The amount of heat released during loading (fig. 14), can be expressed as a function of the increase in the sample temperature from \( T_o \) to \( T \),

\[ q = \Delta u + q_p, \]
where $\Delta m = \bar{\rho} \cdot \varepsilon(T - T_0)$, is the internal energy variation per unit volume; $\bar{\rho}$, $\bar{\varepsilon}$ is the average density, average specific heat, respectively in the sample temperature interval $T_0$...$T$, while $q_0$ is heat per unit volume lost by the sample to the environment.

By experiments has been observed the increase of the loaded sample temperature due to energy dissipation in the case of some metals [33 - 35], rubber – like materials [36] and composite materials [37 - 40].

In the case that the force varies from zero to $F$, one obtains the following expressions for:

- external work per unit volume (curve $\sigma_n(c)(F)$),

$$w_a(F) = \frac{\sigma_n(c)\varepsilon(c)}{(k+1)} = \frac{1}{(k+1)M_n^{1/k}}\sigma_n^{1+1}(c), \quad (106)$$

where $w_a(F) = w_0(c_n) \cdot \sigma_n^{1+1}(c)$.

- internal work per unit volume (curve $\sigma_n(\varepsilon_n)$),

$$w_a(\varepsilon_n) = \frac{\sigma_n(\varepsilon_n)}{(k+1)} = \frac{1}{(k+1)M_n^{1/k}}\sigma_n^{1+1}(\varepsilon_n), \quad (107)$$

The efficiency in solid deformation process, in this case is:

$$\eta_p = \frac{w_a(\varepsilon_n)}{w_a(F)}. \quad (108)$$

**Work and heat when an electric current is passing through a solid conductor**

In the space around a solid conductor with an electric current inside, a magnetic field is formed, in a similar way as in the space around an electrified bar. The electrical and magnetic field play an intermediate role in the process of energy transfer, acting as energy storehouses [41].

The electrical energy developed by the electric current $I$ passing through the conductor, according to the law of energy conservation (26) will be:

$$E_{el} = Q + W_m, \quad (109)$$

where $Q$ is the heat developed inside the conductor, while $W_m$ - magnetic potential energy supplied to the outside or the work potential of the magnetic field ($p_w = 1$). The expressions of the terms from relation (109) depend on the behaviour law in the electric and magnetic field, respectively.

The link between the voltage $U$ and the electric current $I$ passing through a conductor in the case of linear behaviour is given by Ohm’s law, while in the case of the nonlinear behaviour it results from the power law (Table 1).

The electrical resistance in the case given by the power law ($\text{U}=M_R \cdot l^{1/k}$) is:

$$R_n = \frac{dU}{dI} = n_k \cdot M_R \cdot I^{n_k-1} = n_k \cdot (U/I),$$

and it depends on exponent $n_k$ from the behaviour law.

In the case of linear behavior the power supplied to an electrical circuit $P = E \cdot I$, where $E$ – source electromotor voltage, is equal to the thermal power ($U \cdot I$) or heat developed per unit of time, plus the power of the magnetic field $\left(\frac{L \cdot I \cdot dI}{dt}\right)$ or magnetic energy per unit of time:

$$E \cdot I = U \cdot I + L \cdot I \frac{dI}{dt}, \quad (110)$$

where $L$ is the inductance and $t$ - time.

Taking into consideration the balance of powers in eq. (110), the terms in the energy balance (109) are written as:

$$Q = \int U \cdot I \cdot dt; \quad W = \int L \cdot I \cdot dI \quad \text{and} \quad E_{el} = \int E \cdot I \cdot dt,$$

where the intensity of the electric current $I$ is replaced, depending on the material behavior: $I = U/R$ in the case of linear behavior and $I = (U / M_R)^{1/n_k}$ in the case of nonlinear behavior.

By analogy, we do the same in the case of magnetic induction. In the case of linear behavior given by relation $E_m = L \cdot (dI/dt)$, total magnetic energy supplied to the environment is equal to work,

$$W_m = (L \cdot I^2)/2, \quad (111)$$

while the heat quantity is:

$$Q = U \cdot I \cdot t = R \cdot I^2 \cdot t. \quad (112)$$

In the case where there is no electric current in the conductor ($E = 0$), and a magnet is placed near it, an induction current $i$ is produced. Relation (108) becomes:

$$0 = Q - W_m, \quad \text{or} \quad W_m = Q, \quad \text{where} \quad Q = U \cdot i \cdot t.$$

This analysis can be used for improving of losses from pipes or electric cables [42].

The magnetic efficiency of the electric current is

$$\eta_{p,m} = \frac{W_m}{E_{el}}, \quad (113)$$

while the electric efficiency is,

$$\eta_{p,e} = \frac{Q}{E_{el}}. \quad (114)$$

**Numerical examples**

1. A polyethylene melt at $C$ is characterized by the rheological constants:

$$K_n = 4.92 \times 10^9 \text{Pa·s}^{-1} \quad \text{and} \quad \nu = 0.8.$$

It flows through a cylindrical tube featuring a diameter $d = 5 \times 10^{-3} \text{m}$ and a relative length $l = 1/d = 30$. The output is $D_m = 20 \text{kg} / \text{h}$ or $D_v = D_m / \rho = 20 / 920 = 0.021739 \text{m}^3 / \text{h} = 6.038 \times 10^{-6} \text{m}^3 / \text{s}$ where $\rho = 920 \text{kg} / \text{m}^3$ is the melt density.

Calculate the internal power, external power and the power corresponding to melt compressibility.

**a.** The internal power for the flow of a unit volume of melt (96),

$$w_n = \frac{1}{K_n^{1/k} \cdot \nu_{1/k}} = \frac{1}{(4.92 \times 10^9)^{1/8} (735.47 \times 10^3)^{1/8} \cdot 3.8441 \times 10^6} \text{W} / \text{m}^3,$$
where taking into account the Rabinowitsch correction [30; 31], the shear rate
\[ \dot{\gamma} = \frac{3v + 1}{4v} \left( \frac{32}{\pi d^2} \cdot D_0 \right) \left( \frac{32}{\pi (5 \times 10^3)} \cdot 6.038 \cdot 10^4 \right) = 522.7 \text{s}^{-1} \]
and the real shear stress
\[ \tau_r = K^* \cdot \dot{\gamma}^2 = 4.92 \cdot 10^3 \cdot (522.7)^2 \approx 735.47 \cdot 10^5 \text{Pa}. \]

The internal power for the flow through the cylindrical tube
\[ \dot{W}_n = \dot{w}_m \cdot V = 3.8441 \cdot 10^8 \cdot 2.94524 \cdot 10^{-6} = 1.1322 \cdot 10^3 \text{W}, \]
where the melt volume inside the tube
\[ V = \frac{\pi \cdot d^2}{4} \cdot l = \frac{\pi \cdot d^2 \cdot l}{4} \cdot \left( 5 \times 10^3 \right) = 200 \cdot 10^4 \text{m}^3. \]

Consequently, the necessary power for the polyethylene melt flow through the cylindrical tube is approximately \( \dot{W}_n \approx 1.133 \text{ kW} \).

b. The external power for the flow of a unit volume of melt results from the second relation (103)
\[ \dot{W}_e = \left( \frac{dp}{dx} \right)_n \cdot \dot{w}_m = 0.0267 \cdot \dot{w}_m = 1.061 \cdot \dot{w}_m. \]

Where, by comparing the relationships (57) and (58), one obtains
\[ \left( \frac{dp}{dx} \right)_n = \left( 1 + \frac{l_{rel}}{l} \right) \cdot \left( \frac{l}{d} \right) \cdot \left( 1 + \frac{0.8}{30} \right) = 1.0267 \]

where for a polyethylene melt the entrance correction \( l_{rel} / d \approx 0.8 \).

In consequence, the external power for the melt flow through the cylindrical tube,
\[ \dot{W}_e = \dot{w}_m \cdot V = 1.061 \cdot \dot{w}_m \cdot 1.1322 \cdot 10^3 = 1.201 \text{ kW}. \]

c. The power corresponding to the compressibility of melt, taking into account the relationship (81) is
\[ \dot{W}_{cpr} = \dot{W}_e - \dot{W}_n = 1.201 - 1.1322 = 0.0688 \text{ kW}. \]

2. A viscoelastic polymer melt is characterized by the following rheological constants:
\[ K^* = 7.35 \cdot 10^3 \text{ Pa} \cdot \text{s}^* \text{ and } v = 0.5. \]

The correction lengths are: \( l_{rel} / d = 0.9(1 / d) \text{ and } l_{rel} / d = 0.4(1 / d) \)(fig. 9).

Calculate the internal power, external power and the power corresponding to melt compressibility for the flow through a cylindrical tube of diameter \( d = 5 \times 10^{-3} \text{m} \) and relative length \( l_{rel} / d = 10 \). The volume flow rate is \( 6.038 \times 10^{-6} \text{ m}^3 / \text{s} \).

The shear stress in the viscoelastic melt is
\[ \tau_r = K^* \cdot \dot{\gamma}^2 = 7.35 \times 10^3 \times (615.025)^2 = 182.28 \times 10^3 \text{ Pa}, \]
where
\[ \dot{\gamma} = \frac{3v + 1}{4v} \left( \frac{32}{\pi d^2} \cdot D_0 \right) = \frac{3 \times 0.5 + 1}{4 \times 0.5} \left( \frac{32}{\pi (5 \times 10^3)} \times 6.038 \cdot 10^4 \right) = 615.025 \text{s}^{-1}. \]

The internal power results from the second equation (71)
\[ \dot{w}_{m,n} = \frac{1 + \frac{l_{rel}}{l}}{K^*} \cdot \dot{w}_m = \left( \frac{182.28}{(7.35 \times 10^3)^{0.5}} \right) = 1.1211 \times 10^4 \text{ W/m}^3, \]
and
\[ \dot{W}_{m,n} = \dot{w}_{m,n} \cdot V = 1.1211 \times 10^4 \times 98.17477 \times 10^{-6} = 110.064 \text{ W}. \]

In this case, by comparing the relationships (57) and (62) one obtains,
\[ \frac{dp}{dx}_n = \left( 1 + \frac{l_{rel}}{l} \right) \cdot \frac{l}{d} \cdot \frac{l_{rel}}{d} \left( \frac{1 + \frac{0.9}{30}}{1 + \frac{0.4}{30}} \right) = 1.13. \]

Taking into account the second equation (76) one obtains,
\[ \dot{W}_e = \frac{1}{(dp/dx)_n} \cdot \dot{w}_{m,n} = (1.13)^{1.5} \cdot (110.064) = 158.8 \text{ W}. \]

The power corresponding to compressibility and elasticity of the viscous-elastic fluid (81),
\[ \dot{W}_{cpr} = \dot{W}_e - \dot{W}_{m,n} = 158.81 - 110.064 = 48.746 \text{ W}, \]

which represents 30.69% of \( \dot{W}_e \).

3. A sample of carbon steel with the yield stress \( \sigma_y = 350 \text{ MPa} \) is loaded with the normal stress \( \sigma(F) = 300 \text{ MPa} \) calculated based on the traction force \( F \). During the loading from zero to \( \sigma(F) \) the sample heated; the amount of heat per unit volume was \( q = 22 \text{kJ/m}^3 \).

Calculate the error made by neglecting the thermal effect.
At \( \sigma < \sigma_y \) steel behaves linear-elastic.

The external work per unit volume (92):\[ w(\sigma) = \frac{n}{2} \cdot \sigma^2 \]

because \( \sigma^2(\mathbf{c}) = 0 \).
Young modulus \( E = 2.05 \times 10^5 \text{ MPa} \) and \( \sigma^2(\mathbf{c}) = \frac{300 \cdot 2 \times 2 \times 0.5 \times 10^4}{2} = 284.57 \text{ MPa} \).

By neglecting the thermal effect, in this case, the error is
\[ \Delta \sigma = \frac{\sigma(F)}{2E} \cdot \sigma = 300 - 284.57 = 15.43 \text{ MPa} \]
representing 5.14% from \( \sigma(F) \).

Conclusions
In Engineering Thermodynamics textbooks, the analysis of the processes has been carried out usually for thermal agents (materials) in gaseous or vapour state, whose behavior under volume deformation is described by the law of perfect gases or by similar laws (more complicated) for real gases. This is happening because usually in Thermal Machines only these sort of thermal agents are used (gases and vapour). In some cases like in the Rankine cycle (the cycle of the Powerplants) and in refrigeration machines also the liquid state is considered. Because of this, the most Engineering Thermodynamics textbooks, refer only to the gas and vapour volume deformation processes (like: isothermal, isobaric, adiabatic, politropic.)
The paper analyzes the energy aspects of thermo mechanical processes, specific both to gases and viscous and viscoelastic fluids, as well as to thermo mechanical and thermo electromagnetic ones in solid bodies. On the Energonics bases and its concepts, energy and work are correlated with the matter behavior. One considers both the linear as well as the nonlinear behavior of the solids and fluids under analysis for which there have been deduced the expressions of heat and work, systematized in table 4. One of the contributions of this paper lies in extending the thermodynamic analysis to fluid flow, to solid body deformation and to thermo electromagnetic processes in solid bodies, in the spirit of Energonics.

In all the cases under analysis, the external load and its effect have been decomposed, from the following points of view:

Table 4

<table>
<thead>
<tr>
<th>Thermodynamic Process</th>
<th>Behavior law</th>
<th>Work $W$</th>
<th>Heat $Q$</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Isometric (V = constant)</td>
<td>$pV = \frac{T}{T_0}$</td>
<td>$W = 0$</td>
<td>$Q = \frac{m}{c_p} (T - T_0)$</td>
<td>$n = \infty$</td>
</tr>
<tr>
<td>2. Isothermal (T = constant)</td>
<td>$pV = \frac{T}{T_1}$</td>
<td>$W = \int (VV_0 - V_0V) dt$</td>
<td>$Q = \frac{m}{c_p} (T - T_0)$</td>
<td>$n = 0$</td>
</tr>
<tr>
<td>3. Adiabatic (Q = 0)</td>
<td>$pV = \frac{T}{T_1}$</td>
<td>$W = \frac{pV}{k-1}$</td>
<td>$Q = \frac{m}{c_p} (T - T_0)$</td>
<td>$n = k$</td>
</tr>
<tr>
<td>4. Polytropic (n = constant)</td>
<td>$pV = \frac{T}{T_0}$</td>
<td>$W = \frac{pV}{k-1}$</td>
<td>$Q = \frac{m}{c_p} (T - T_0)$</td>
<td>$n = \frac{k}{k-1}$</td>
</tr>
</tbody>
</table>

b. Perfect gases in irreversible processes.

6. See Table 3 (1 – 5)

| Energy aspects of thermo mechanical processes, specific both to gases and viscous and viscoelastic fluids, as well as to thermo mechanical and thermo electromagnetic ones in solid bodies. |

| Table 4 |

| WORK (PROCESS MECHANICAL ENERGY EXCHANGED) AND HEAT (PROCESS THERMAL ENERGY EXCHANGED), depending on the particularities of the process in the case of the perfect gases in reversible processes (1 – 5) or in irreversible processes (6 – 10), of viscous (11 and 12), viscoelastic (13 and 14) liquids and of solids (15 – 17). |

| 11. | $\tau = \mu \cdot \dot{\gamma}$ | $W = \frac{1}{\mu} \int \dot{\gamma} \cdot V \cdot t$ | $Q = -W$ | $\frac{\mu \cdot \gamma}{\mu}$ |
| 12. | $\tau = K_1 \cdot \dot{\gamma}$ | $W = \frac{1}{K_1} \int \dot{\gamma} \cdot V \cdot t$ | $Q = -W$ | $\frac{\tau \cdot \gamma}{\mu}$ |
| 13. | $\tau = \sigma \cdot \dot{\varepsilon}$ | $W = \frac{1}{\sigma} \int \dot{\varepsilon} \cdot V \cdot t$ | $Q = -W$ | $\frac{\tau \cdot \gamma}{\mu}$ |
| 14. | $\tau = K_e \cdot \dot{\gamma}$ | $W = \frac{1}{K_e} \int \dot{\gamma} \cdot V \cdot t$ | $Q = -W$ | $\frac{\tau \cdot \gamma}{\mu}$ |
| 15. | Polytropic | $W = \frac{m}{c_v} (T - T_0)$ | $Q = \frac{m}{c_v} (T - T_0)$ | $n = \frac{k}{k-1}$ |
| 16. | Adiabatic (Q = 0) | $W = \frac{m}{c_v} (T - T_0)$ | $Q = \frac{m}{c_v} (T - T_0)$ | $n = \frac{k}{k-1}$ |
| 17. | $U = R_1 \cdot I$ | $W = \frac{1}{L} \int I^2 \cdot t$ | $Q = \frac{R_1}{1} \cdot I^2$ | $t$-time; $L$-inductance. |

Observation: $\varepsilon_\gamma$; $\varepsilon_c$; $\tilde{\varepsilon}$ - mean specific heat in the temperature interval $T_i$, $T_f$ under constant pressure, constant volume, independent of volume and the pressure, respectively; $\Delta U$ - enthalpy variation.
- mathematical, by expressing the total tensor as a sum of two particular tensors;
- physical, by expressing the energy of external action as a sum of shape variation energy and volume variation energy;
- thermodynamic, by equating the external action energy as the sum between the internal energy variation and mechanical work, using the First Law of Thermodynamics.

By correlating the behavior laws of solids, fluids and viscos and viscoelastic fluids respectively with the fundamental definitions of work and Power respectively, taking into consideration the principle of energy conservation, there have been obtained relations for:
- external work or work of external action;
- internal work or Power corresponding to the effect produced upon the body under the load.

One has found the analogy between the processes to which perfect gases are subject to, in ideal (reversible) and in real (irreversible) processes, for ideal gases, solids and viscoelastic liquids and viscoelastic fluids, which allows one to deal with a unitary approach from a Thermodynamics point of view. In Table 5 there have been included the parameters characteristic of thermodynamic analysis of perfect gases, solids, viscous and viscoelastic fluids.

The behavior laws take into consideration the reluctance or the opposition of material to the change in the state of momentary equilibrium \[\text{(43; 44)}\], in a different manner for gases, liquids and solids, namely:
- with gases, only their volume variation depending on pressure and temperature, \[V = \text{f}(p; T)\], by neglecting the flow effect (\(\mu = 0\)). One considers only the volume deformation effect, taking into account the influence of process speed and the direction of deformation (expansion or compression);
- with solids, through the dependence of applied normal stress on strain and of electrical voltage on electric current intensity. The matter constants (E, M, k as well as L, R, M, n) depend on temperature and some of them on the process speed;
- with viscous fluids (fluids), through the dependence of applied shear stress on the shear rate. Fluid constants (\(\mu, K, \nu\) and \(\mu, K, \nu\)) depend on temperature;
- with viscoelastic liquids (fluids) through the dependence of shear and normal stress on the shear strain rate and strain rate, respectively. The fluid constants (\(\sigma, \gamma, \nu\) respectively) depend on temperature (Table 1).

The expressions obtained for work (process mechanical energy exchange) in the case of processes in perfect gases, viscous liquids, viscoelastic liquids and solids are, formally, similar and depend on process parameters, as well as on the material behavior in the process conditions, by means of the exponent from the behavior law.

Heat (process thermal energy exchange) depends on the process parameters and on constants deriving from the behavior law.

The analysis carried out by considering the concept of Energy and the Principle of Energy Conservation (relations \((26) - (29)\) and \((31)\), has made it possible to deduce correctly relations for work, heat and process efficiency \((50)\) involving gases, liquids or solids. Diagrammes p - V (fig. 4), τ(p) - γ (fig. 10) and \(\sigma, / v\) (fig. 14) emphasize the correlation between process parameters which determine the external work and the process parameters which determine internal work and heat; in the case of viscoelastic fluids, one also discriminates the elastic effect (fig. 10) which determines the Power corresponding to the relaxation of elastic stresses upon leaving the nozzle.

### References

43. JINESCU VV., Rev. Chim. (Bucharest), 34, 1983, p. 491

Manuscript received: 24.10.2012