Mathematical Model for Determining the Volume of Abrasive Susceptible of Levitation

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On the background of grinding processes and of the practical appliances of magnetic fluids there have been developed manufacturing technologies that use rheological abrasive media. One of the forms accomplished and developed by the rheological abrasive media are the magneto-rheological abrasive media. These media are based on the capacity of the magnetic field to act upon some abrasive grains with a force higher enough to compact them and create abrasive actions. There is used a chemical solution - a mixture of magnetic field, magnetic fluid and non-magnetic abrasive materials, which, according to the principle of magnetic levitation induce ascending forces which can be used as splintering forces. The action of the magnetic field on the abrasive grains consists mainly in the concentration and leading of the abrasive grains on the manufactured area. The energy of the magnetic field and the relative movements in the process determine a mechanical splintering. The authors propose in this paper to determinate the relationships between the number of grains, those dimensions and the pressure developed in magneto-rheological abrasive media.

Keywords: modeling, levitation, rheological abrasive medium

According to the principle of the magnetic levitation, an increasing force called levitation force acts upon a non-magnetic body that has been introduced in an activated magnetic fluid [1-2].

In case of the splintering processes with magneto-rheological abrasive mediums, the levitation forces are dependent directly on the volume of the abrasive grains that exist in the magnetic fluid [3]. This concentration (c) is defined as the ratio of the abrasive quantity (ma) to the total quantity of abrasive suspension (magnetic fluid (ml) and abrasive (ma))

\[ C = \frac{m_a}{m_a + m_l} \]  

Expressing these quantities according to the corresponding densities and volumes, we obtain [4]:

\[ C = \frac{m_a}{m_a + m_l} = \frac{\rho_a \cdot V_a}{\rho_a \cdot V_a + \rho_l \cdot V_l} \]  

From (2) we express the abrasive volume to concentration:

\[ V_a = \frac{\rho_l \cdot V_l \cdot C}{\rho_a \cdot (1 - C)} \]  

Sure, the change of the dimension of the abrasive grain determines the change of volume of the side surface and its perimeter. To express analytically these relations, the following assumptions are made [5]:
- the volume of the abrasive \( V_a \) increases with the increase of the abrasive grain dimension, \( g_a \);
- by the decrease of the dimension \( g_a \) the number of grains in the volume of the abrasive is increased as well as the number of splintering peaks;
- the increase of the number of abrasive grains leads to the increase of the contact surface between the abrasive material and the magnetic fluid;

-there are used abrasive grains with dimensions \( g_a \) which make cvasi-geometrical progression as well as of the rou now of normalized numbers.

There appears the necessity to introduce a \( C_g \) geometrical coefficient to express the variation of splintering geometrical elements (peaks, edges, surfaces) in relation with the dimension of abrasive material; this coefficient will affect the volume of levitating abrasive.

Analytical model

Consider an elementary volume of abrasive material, cube shaped with a side \( a = 1 \text{ mm} \). The volume, the surface and the length edges of this cube will be:

\[ V_a = a^3 \quad S_i = 6 \cdot a^2 \quad L_i = 12 \cdot a \]  

Dividing each side of the elementary cube in 2 equal parts (fig.1), we obtain 8 equal cubes, with a side half of the initial one, \( a/2 \).

Similarly, the side of this cube can be divided to 3, 4, 5, …, equal parts. Suppose we divided the cube in “i” parts, Fig.1. The cube associated to the abrasive grain
we obtain “i³” cubes with a/i sides, with the volume, total area and edge lengths given by the following:

\[ V_i = \frac{V_i}{i^3} \quad S_i = 6 \left( \frac{a}{i} \right)^2 \quad L_i = 12 \left( \frac{a}{i} \right) \quad (5) \]

Summing up the volumes, total areas and edge lengths of i³ we obtain:

\[ V_i = V_i \cdot i^3 = \frac{V_i}{i^3} \cdot i^3 = V_i \quad (6) \]

\[ S_i = S_i \cdot i^3 = 6 \cdot \left( \frac{a}{i} \right)^2 \cdot i^3 = 6 \cdot a^2 \cdot i = S_i \cdot i \quad (7) \]

\[ L_i = L_i \cdot i^3 = 12 \left( \frac{a}{i} \right) \cdot i^3 = 12 \cdot a^2 \cdot i = L \cdot i^2 \quad (8) \]

By forming the i³ cubes the number of splintering peaks has increased:

\[ \nu_{fr} = \nu_{i,i} \cdot i^3 \quad (9) \]

Finally, for an elementary volume V₁ divided in i³ equal parts, the number of splintering peaks increases with i³, the length of edges with i² and the side area with i.

The Cg, geometrical coefficient will follow these effects:

\[ C \equiv f (\text{volume, surface, length, peaks}) \]

The analytical expression of this geometrical coefficient will be:

\[ C_g = b_0 + b_1 \cdot i + b_2 \cdot i^2 + b_3 \cdot i^3 \quad (10) \]

From (6), (7), (8) and (9) we obtain:

\[ \frac{V_i}{V_i} = 1 \quad \frac{S_i}{S_i} = i \quad \frac{L_i}{L_i} = i^2 \quad \frac{\nu_{fr}}{\nu_{fr}} = i^3 \quad (11) \]

By these values the (10) becomes:

\[ C_g = b_0 + b_1 \cdot i + b_2 \cdot i^2 + b_3 \cdot i^3 \quad (12) \]

Expression (12) represents a cubic parabola where polynomial coefficients can be determined by the limit condition applying the continuous and derivable polynomial functions, or by the smallest squares method [6-7].

In the first case we make the following remarks:

- if V₁ = 1 mm³, the i variable represent the reverse of granulation, gₐ, respectively the numbering of grains on the length unit, will establish M = (0 … 1000) as interval in the study of polynomial function;

- as a function of level 3, sure at least one alternation of sign on R (-∞, + ∞) and how the geometric shape factor C (g) is positive, c(0) = b₀ ⇒ b₀ > 0;

- function is not injective, so there are two x₁ ≠ x₂ values for which f (x₁) = f (x₂);

- the function admits at least one extreme;

- the theory of Fermat and the theory of Rolle will be applied on this continuous interval of the function, admitting a relative extreme, so:

\[ \exists \ a \in M \text{ such that } C'(a) = 0 \]

\[ \exists \ i \in M \text{, for which } C'(i) = 0 \]

\[ \forall \ b, b_1, b_2, b_3 \in R \text{ such that } C(a) = C(b) \]

\[ \exists \ i \in M \text{, for which } C'(i) = 0 \]

\[ \exists \ i \in M \text{, for which } C''(i) = 0 \]

The system can be solved by several methods (Simplex, digital, linear programming, etc.).

Using simplex method were first obtained the following values:

\[ b_o = 800561.4 \cdot 10^{-5} \; ; \; b_1 = -33783.3 \cdot 10^{-5} \; ; \; b_2 = -1031.7 \cdot 10^{-5} \; ; \; b_3 = 1 \cdot 10^{-5} \]

In addition, it was performed and a 2² factorial experiment which confirmed the values analytically obtained.

Factorial experiment had the light response of cutting pressure. This pressure cutting abrasive environment that carries a magneto-rheological on the treatment was determined using an experimental stand (fig.2) whose design has considered the following requirements:

- direct convert pressure into electrical signals easily processed;

- providing a high speed response under conditions of high sensitivity;

- creation of a rigid system of measurement.

The expression of the geometric coefficient becomes:

\[ C_g = 800561.4 \cdot 10^{-5} - 33873.3 \cdot 10^{-5} \cdot i - 1031.7 \cdot 10^{-5} \cdot i^2 + 10^{-5} \cdot i^3 \quad (13) \]

Fig.2. Experimental stand

\[ \Delta = 4(b^2 \cdot b_2^2 - 12b \cdot b_3 > 0 \]

\[ b_3 > 0 \text{ and } b_3 < 0 \]

- order derivative II is: C'' = b₁ = 2b₂ . i + 3b₃ . i² must be negative, because the function is concave and inflection should be in the positive, the roots of the derivative I

\[ \frac{-2b_3 - \sqrt{4b_2^2 - 12b_1 \cdot b_3}}{2} < b_2 < \frac{-2b_3 + \sqrt{4b_2^2 - 12b_1 \cdot b_3}}{2} \]

All these relationships lead to the system:

1. \[ b_3 > 0 \]
2. \[ 4(b^2) - 12b_1 \cdot b_3 > 0 \]
3. \[ b_3 > 0 \]
4. \[ b_2 < 0 \]
5. \[ \frac{-2b_3 - \sqrt{4b_2^2 - 12b_1 \cdot b_3}}{2} < b_2 < \frac{-2b_3 + \sqrt{4b_2^2 - 12b_1 \cdot b_3}}{2} \]

6. \[ \exists \ a \in M, b \in M \text{ such that } C'(a) = C'(b) \]
7. \[ \exists \ i \in M, \text{ for which } C'(i) = 0 \text{ and } C''(i) = 1 \]

The system can be solved by several methods (Simplex, digital, linear programming, etc.).

Using simplex method were first obtained the following values:

\[ b_o = 800561.4 \cdot 10^{-5} \; ; \; b_1 = -33783.3 \cdot 10^{-5} \; ; \; b_2 = -1031.7 \cdot 10^{-5} \; ; \; b_3 = 1 \cdot 10^{-5} \]

In addition, it was performed and a 2² factorial experiment which confirmed the values analytically obtained.

Factorial experiment had the light response of cutting pressure. This pressure cutting abrasive environment that carries a magneto-rheological on the treatment was determined using an experimental stand (fig.2) whose design has considered the following requirements:

- direct convert pressure into electrical signals easily processed;

- providing a high speed response under conditions of high sensitivity;

- creation of a rigid system of measurement.

The expression of the geometric coefficient becomes:

\[ C_g = 800561.4 \cdot 10^{-5} - 33873.3 \cdot 10^{-5} \cdot i - 1031.7 \cdot 10^{-5} \cdot i^2 + 10^{-5} \cdot i^3 \quad (13) \]
All these theories prove the fact that the volume of levitating abrasive material is influenced by concentration \( C \) and the number of abrasive grains per unit of length \( i \) (the reverse of granulation, \( g_a \)). The analytical expression of abrasive volume susceptible to levitate on the working piece surface will be:

\[
V_s = V_s \cdot C_g = \frac{\rho_s \cdot V_s \cdot C}{\rho_a \cdot (1-C)} = f(C, i) \quad (14)
\]

After all the replacements we shall obtain:

\[
V_s(C, i) = \frac{10^{-7} \rho_s \cdot V_s \cdot C}{\rho_a \cdot (1-C)} \quad (800561.4 - 33873.3 \cdot i - 1031.7 \cdot i^2 + i^3)
\]

This expression represents the explicit form of a function \( V_s(C, i) \) with two variables \((C, i)\).

The splintering pressure on the manufacturing surface is proportional to this volume. After all the replacements we obtain the splintering pressure:

\[
V_s(C, i) = \frac{10^{-7} \rho_s \cdot V_s \cdot C}{\rho_a \cdot (1-C)} \quad (800561.4 - 33873.3 \cdot i - 1031.7 \cdot i^2 + i^3)
\]

Figure 3 represents the shape of the answering surface \( p = f(C, i) \).

Figure 4 represents the curves of constant level obtained by splinting the answering shape \( p = f(C, i) \) with horizontal surfaces.

The analysis of the answering surface \( p = f(C, i) \) described in (16) and drawn on figure 3 and figure 4, leads to the following remarks:

- the splintering pressure calculated by relation 16 have values between 0.02 ... 0.50 MPa, and are placed in the range of values recommended for the finishing processing (at honing \( p = 0.03 \ldots 0.40 \) MPa, at superfinition \( p = 0.05 \ldots 0.6 \) MPa and at lapping \( p = 0.05 \ldots 0.45 \) MPa);
- the independent variables, the concentration \( C \) and granulation \( g_a \), expressed by the number of grains \( i \) per length unit varies in the limit of technological interest;
- according to the number of grains \( i \) per length unit, the answer surface represents a local extreme of maximum type, which demonstrates that theoretically, the hypotheses made for mathematical model are correct and describe the real phenomena of the process;
- according to concentration \( C \) of the magneto-rheological abrasive solution, the answer surface does not present extreme. This means that the hypotheses could be considered simplifying, apart from the physico-chemical phenomenon in the solution or that theoretically, the extreme is not in the technological interest area for this variable. There is also a third motivation for this aspect, which is more plausible that the levitation phenomenon at the individual abrasive grain level differs from that at the sublevel of grain and of the whole solution. During the manufacturing some turbulences occur and the hydrodynamic flow regime differs from the static one.

Conclusions

The adequacy of this analytical model for magneto-rheological abrasive solutions can be studied by the accomplished experiences according to corresponding schedules.

References

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