Influence of the Losses by Reflection on the Channeled Spectrum of a Nematic Liquid Crystalline Layer

IRINA DUMITRASCU*, LEONAS DUMITRASCU, DANA-ORTANSA DOROHOI
“Al.I. Cuza” University, Faculty of Physics, 11 Carol I Bdv., 700506, Iasi, Romania

A simulation of the transmission factor of a nematic liquid crystalline layer (MBBA) kept in a special cell placed between two crossed polarizers is presented in this paper. The equations of the mathematical model contain the influence of the losses by reflection on the transmission factor of the device. The losses by reflection are estimated on the basis of Fresnel coefficients for normal incidence. Modifications induced by reflection losses in the channeled spectra are dependent on the main refractive indices both of the nematic liquid crystalline layer and of the glass walls of the cell.

Keywords: MBBA, birefringence, losses by reflection, transmission factor of liquid crystalline layer between crossed polarizers, channeled spectrum

The magnitude of the electric components of the reflected (E_r) and transmitted (E_t) electromagnetic waves at the separation surface of two transparent media depend on their refractive indices n_1 and n_2. At normal incidence these components are proportional with Fresnel coefficients [1-4]:

\[
\frac{E_r}{E_i} = \frac{n_2 - n_1}{n_1 + n_2}, \quad \frac{E_t}{E_i} = \frac{2n_2}{n_1 + n_2}
\]

In relations (1) \( E_i \) represents the electric component of incident wave, parallel to the separation surface. From relations (1) it results that the magnitude of the electric component of the incident wave is modified by reflection and transmission. The sense of the incident and transmitted electric components is the same, indifferently on the values of n_1 and n_2.

In the case of a separation surface between one isotropic and one anisotropic media, the transmitted wave in the anisotropic medium can be thought as being decomposed in the linearly polarized waves (acting on the main directions) which propagates with different velocities. The ordinary and extraordinary waves keep their polarization state in the propagation process. For a given direction of propagation the two directions are perpendicular and parallel to the basically vibration directions of the anisotropic medium.

Due to the difference between the main refractive indices of the anisotropic medium, \( n_o = n_{o_1}, n_e = n_{e_1} \) and \( n_o \neq n_e \), the two transmitted linearly polarized components, suffer saltus in magnitudes in different proportions, both at the entrance and at exit of the anisotropic layer. These modifications influence the theoretical expression of the transmission factor of the devices containing an anisotropic layer between two crossed polarizers. This study is of a great interest in the analysis of the channeled spectrum in optical range and also when tunable filters are projected.

The aim of this paper is to evidence the diminishing of the emergent radiant intensity from the optical devices due to the losses by reflection at the separation surfaces. A simulation of the transmission factor of a device consisting from a liquid crystalline layer (MBBA) between two crossed polarizers is made for two cases: when the losses by reflection at the separation surfaces are neglected and when they are considered. The transmission factor of the considered device for the first case is that established in the literature [3, 4] and the transmission factor in the second case is established below.

Theoretical notions

Let us consider an electromagnetic, linearly polarized, plane wave which enters under normal incidence in a device consisting of an anisotropic layer (CL), between two crossed polarizers (P and A).

The liquid crystalline layer (CL) is kept in a cell with glass walls (St_1 and St_2). Let be L the thickness CL of and the thicknesses L_1 and L_2 of the glass walls (fig. 1).

Let be the magnitude of the linearly polarized electric component in the transmitted wave after the first polarizer [1-3]:

\[
E = E_o \cos(\omega \cdot t + \phi_o)
\]

After the first glass wall of the cell containing CL, this component becomes:

\[
E = \frac{2}{(n_1 + 1)} E_o \cos(\omega \cdot t - k_o n_1 L_{s1} + \phi_o)
\]

In relation (3) \( n_o = 1 \) and \( n_o \) is the refractive index of

![Fig. 1. Device used for channeled spectrum obtaining](http://www.revistadechimie.ro)
Before the entrance in CL, the electric components of the two considered linearly polarized components acting parallel to the main directions of the anisotropic layer are of the type (fig. 3):

\[
\begin{align*}
E_o &= \frac{2}{(n_1 + 1)(n_1 + n_2)} E_o \cos \theta \\
E_e &= \frac{2}{(n_1 + 1)(n_1 + n_2)} E_o \sin \theta
\end{align*}
\]

The optical phenomena at the separation surface between the first glass wall of the cell and CL modify differently the magnitudes of ordinary (oa) and extraordinary (oc) components. These modifications are described by Fresnel formulae [1, 2]:

\[
\begin{align*}
E_{oa} &= \frac{4 n_a}{(n_1 + 1)(n_1 + n_2)} E_o \cos \theta \\
E_{oc} &= \frac{4 n_e}{(n_1 + 1)(n_1 + n_2)} E_o \sin \theta
\end{align*}
\]

In relations (5), \( n_a \) and \( n_e \) are the main refractive indices of CL.

At the exit from the anisotropic CL layer, the magnitudes of the two electric components are also modified, so they become:

\[
\begin{align*}
E_{oa2} &= \frac{8 n_a n_{oa}}{(n_1 + 1)(n_1 + n_2)^2} E_o \cos \theta \\
E_{oc2} &= \frac{8 n_e n_{oc}}{(n_1 + 1)(n_1 + n_2)^2} E_o \sin \theta
\end{align*}
\]

After the second glass wall and after the second polarizer (A) the magnitudes of the electric components of the transmitted wave (fig. 4) are given by relation (7):

\[
\begin{align*}
E_{oad} &= \frac{16 n_2^2 n_{oa}}{(n_1 + 1)(n_1 + n_2)} E_o \cos \theta \cos \beta \\
E_{ocd} &= \frac{16 n_2^2 n_{oc}}{(n_1 + 1)(n_1 + n_2)} E_o \sin \theta \sin \beta
\end{align*}
\]

The phases of the two components at the entrance of the second polarizer can be written as:

\[
\begin{align*}
\varphi_a &= \varphi_e - k_o n_{oa} L - k_e n_{oc} (L_{21} + L_{22}) \\
\varphi_e &= \varphi_a - k_o n_{eo} L - k_e n_{ee} (L_{21} + L_{22})
\end{align*}
\]

The phase difference between the two linearly polarized components is expressed in (8).

\[
\Delta \varphi = \varphi_e - \varphi_a = k_o L (n_a - n_e) = \frac{2 \pi}{\lambda} (n_a - n_e) L
\]

At the exit from the second polarizer, the electric component of the electromagnetic wave (fig. 4) is the vectorial sum of the projections of ordinary and extraordinary components on the transmission direction of polarizer A:

\[
E_A = E_{oad} + E_{ocd}
\]

When the two polarizers of the device are crossed, the angle \( \beta \) satisfies the condition:

\[
\beta = \theta + \frac{\pi}{2}
\]

The magnitude of the electric component of the resultant wave after the polarizer A becomes:

\[
E_{oad} = \frac{8 n_2^2}{(n_1 + 1)^2} E_o \sin \theta \sqrt{\frac{n_a^2}{(n_1 + n_2)^2} + \frac{n_e^2}{(n_1 + n_e)^2} - \frac{2 n_a n_e}{(n_1 + n_2)(n_1 + n_e)} \cos \Delta \varphi}
\]

The last relation indicates that the magnitude of the electric field intensity of the resultant wave does not become null indifferent on the frequency of the incident wave and its values are between the extremes \( E_{oa A min} \) and \( E_{oa A max} \):

\[
E_{oad} \leq E_{oad} \leq E_{oad A max}
\]

In (13), the following notations were made:

\[
\begin{align*}
E_{oad min} &= \frac{n_a}{(n_1 + n_2)^2} \frac{n_a^2}{(n_1 + n_e)^2} \\
E_{oad max} &= \frac{n_e}{(n_1 + n_2)^2} \frac{n_e^2}{(n_1 + n_e)^2}
\end{align*}
\]

Relations (13) and (14) show that the transmission factor of the device \( D \) used for channeled spectra obtaining has the minima of channeled differing to zero.

Relation (15) gives the theoretical expression of the transmission factor of the device for the case \( \theta = 45^o \) (fig. 2):

\[
T(\lambda_0) = \frac{1}{2} \left( \frac{E_{oad}}{E_o} \right)^2 = \frac{32 n_2^2}{(n_1 + 1)^2} \left( \frac{n_a^2}{(n_1 + n_2)^4} + \frac{n_e^2}{(n_1 + n_e)^4} - \frac{2 n_a n_e}{(n_1 + n_2)(n_1 + n_e)} \cos \Delta \varphi \right)
\]
Experimental data used for simulation

The experimental data used for simulation (fig. 5) from papers [5-7] refer to N-(4-methoxybenzilidene)-4-butylaniline liquid crystal (MBBA).

![Fig. 5. MBBA main refractive indices vs. wavelengths of visible radiations in absence and in presence of an electric external field applied between the internal cell walls](image)

The dispersion of the birefringence can be represented as a Cauchy relation [5]:

$$\Delta n(\lambda) = A_i \frac{\lambda^2}{\lambda^2 - \lambda_i^2} + A_i \frac{\lambda^2}{\lambda^2 - \lambda_i^2}$$  \hspace{1cm} (16)

The fitting coefficients $A_i$ $(i=3)$ in (16) depend on the external electric field applied to the MBBA layer (fig. 5). In the first approximation, for a linear dependence between the external electric field and the birefringence, Cauchy coefficients can be written as in (17) [5, 6]:

$$A_i(E) = A_i(0) + \alpha_i \cdot E, \quad (i = 1, 3)$$  \hspace{1cm} (17)

The fitting coefficients $A_i$ and $\alpha_i$ $(i=3)$ from table 1 were obtained by using the data from figures 5 and 6, based on relations (16) and (17).

**Table 1**

<table>
<thead>
<tr>
<th>$i$</th>
<th>$A_i(0)$</th>
<th>$\alpha_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.5850\times10^{-2}</td>
<td>8.6929\times10^{-6}</td>
</tr>
<tr>
<td>2</td>
<td>5.0536\times10^{-15}</td>
<td>9.3874\times10^{-21}</td>
</tr>
<tr>
<td>3</td>
<td>5.5847\times10^{-28}</td>
<td>3.8285\times10^{-34}</td>
</tr>
</tbody>
</table>

The glass dispersion can be calculated with a good precision in the visible range, by using Cauchy formula (16). The fitting Cauchy coefficients for glass [8] have the following values: $A_1 = 1.4580$ and $A_2 = 3.54 \cdot 10^6 \text{m}^2$ and $A_3 = 0$.

**Results and discussions**

In figure 6 and 7 the wavelength dependence of the transmission factor of the device $D$ for a thickness 14 $\mu$m of the liquid crystalline layer is given. The simulated channeled spectrum was obtained with the birefringence data from [5, 6]. Figure 6 corresponds to the absence of the external electric field and figure 7 corresponds to an external electric field of about $10^5 \text{V/m}$, applied between the internal walls of the cell containing liquid crystal.

The transmission factors of the device from figure 1 computed when the reflection losses are neglected are given in figures 6 and 7 by fine lines, in order to evidence the importance of considering them.

It result from figures 6 and 7 that the transmission factor is affected by significant diminutions – from 0.5 to 0.18 – determined by the modifications of the refractive indices of the media from which the device is made. From these figs., it results that the number of the channels increases in the presence of one external electrostatic field, due to the increase of the liquid crystal birefringence, by ordering action on the mesogenic molecules.

The ordering action of an external electric field was evidenced in various papers [9-16] and applications were proposed [16-19] on the bases of these studies.

**Conclusions**

The losses by reflection influence significantly the maxima of the transmission factor of the device. The external electric field applied between the internal walls of the cell containing the liquid crystal, insignificantly influences the magnitudes of the maxima in channeled spectra, but the number of channels increases due to the presence of the external electrostatic field, proving its ordering action on the nematic liquid crystal molecules.

**References**

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