An Algorithm for the Design of Selection Pipes and Process Parameters in the Decanting of Solid-Liquid Heterogenous Systems by Levigation

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We propose a novel general algorithm for the design of the installation and computation of the process parameters involved in granulometric class sorting by levigation of a granular non-homogenous material. We present a mathematical approach to derive the algorithm, as well as practical conditions and restrictions – some of which novel – used in this derivation. We discuss both continuous (single step) as well as intermittent (multiple step) levigation. The theoretical algorithm has been experimentally confirmed in the laboratory using a pilot installation; the results of the experimental measurements have been compared with previously obtained measurements that used alternative derivations.

Keywords: sorting, backwash sedimentation, levigation installation.

There are situations in both industrial practice as well as experimental research where there is need for dimensional sorting as well as separation in granulometric classes of granular heterogeneous materials. There is a rather large number of methods used for this purpose, each applicable to a certain domain and requiring certain techniques [1-5]. We will focus on the levigation method also known as the elutriation method (using the elutriator) [2], or backwash sedimentation. This method is applicable in scenarios where Stokes’ law is valid: it involves the dispersion of the granular material in a liquid, resulting in the suspension of the material. Water is most often used as liquid environment, and the dimension range of the particles to be sorted is approximately (1, 1000) μm.

Consider an amount \( m \) of a granular material composed by approximately spherical solid particles of various diameter \( d \) values. The material is dispersed in a liquid placed inside a circular vertical cylinder of diameter \( D \), height \( H \), volume \( V \), and constant transversal section \( S \) (fig. 1).

Following Stokes’ law, a solid spherical particle placed in a liquid at rest falls with a speed \( v \) proportional to the square of its diameter \( d \), that is:

\[
v = K \cdot d^2 ,
\]

where \( K \) denotes:

\[
K = \frac{\rho_s - \rho_w}{18 \cdot \eta_w} \cdot g
\]

and \( g \) – gravitation acceleration, in m/s²; \( \rho_s, \rho_w \) - solid and liquid phase densities, respectively, in kg/m³; \( \eta_w \) - dynamic viscosity of the liquid in kg/(m·s) or N·s/m².

The parameters \( \rho_s, \rho_w, \eta_w \) are material constants and thus \( K \) is a constant as well with value depending on the particular suspension (granular material – liquid pair).

Assume the liquid environment of the suspension (continuous phase) moves upward with constant speed \( v_w \); then the absolute speed \( v_a \) – relative to the pipe’s walls – of a particle of diameter \( d \) is:

\[
v_a = v_w - v .
\]

where:

\[
v_w = \frac{4Q}{\pi D^2} .
\]

Note that for certain combinations of values for \( Q \) and \( D \) there is a value of \( d_L \) of the diameter \( d \) for which \( v_a = 0 \). Particles of diameter \( d > d_L \) exhibit \( v_a < 0 \) and thus move downward and decant at the basis of the cylinder (for example, particle A in figure 1), while particles of diameter \( d < d_L \) have \( v_a > 0 \) and thus move upward and are evacuated from the cylinder at its upper extremity (for example, particle B in fig. 1). As a result, solid particles which are initially in suspension are separated in two granulometric classes: a class with particles of diameter \( d > d_L \) and another class with particles of diameter \( d < d_L \). We call the value \( d_L \) as the limit between the two granulometric classes.

We now generalize this concept. Separation of a solid
material in suspension in $N$ granulometric classes delimited by $N-1$ values of $d_i$ denoted in decreasing order $d_1, d_2, \ldots, d_N$, can be achieved using $N - 1$ cylinders (called selection pipes or elutriators); each of the cylinders has a particular value for the parameters $Q$ and $D_i$ corresponding to one of the $d_i$ values. Separation by levigation will result in granulometric class $i$ (formed by particles of diameter $d_i$, $d_i < d < d_{i+1}$) being collected at the basis of cylinder $i$ ($i=1, N-1$) the particular values $d_i$ and $d_1$ denote $d_i \equiv d_{\max}, d_1 \equiv d_{\min}$; ($d_i \geq d_{\max}, d_{\min} \geq d_1$) where $d_{\max}$ and $d_{\min}$ are the extreme values of the particles diameter in the material to be separated (fig. 2).

The limit values $d_i, d_1$ ($i=1, N-1$) are usually either standardized by granulometric scales or set by practical requirements. This requires a corresponding design of both the dimensions of the levigation installation (the diameter and height of each pipe) and of the technological process (the liquid flow of the dispersion continuous phase, the process duration and possibly certain characteristics of the liquid such as density and viscosity).

**Algorithm for dimensions fit and design**

**Diameter and flow of the selection pipes**

We analyze two possible approaches, which differ in the number of steps required for decanting completion.

The first approach is one-step (or continuous) levigation in which all selection pipes are connected in series with the number of steps required for decanting completion. The continuous levigation, in which each step involves sorting a single granulometric class in decreasing order with the dimension of composing particles (starting with the class of largest particles and ending with the class of smallest particles). In each step the installation uses a single selection pipe but the diameter of the pipes as well as the flow change from one step to the next with values corresponding to the granulometric class to be separated. Thus, step $i$ ($i=1, N-1$) separates granulometric class $i$ formed by particles of diameter $d_i \geq d$ from the rest of granulometric classes with particles of diameter $d < d_i$.

As the continuous levigation, restrictions are valid with this approach as well, while the diameters and flow are also computed using (6), (7).

**Heights of the selection pipes**

The heights $H_i$ of the selection pipes are first lower bounded by the condition that the volume $V_i$ of the cylinder needs to exceed a certain minimum value $V_{\text{max}}$, determined such that the concentration $c$ of solid particles dispersed in the liquid be smaller than a maximum admissible value $c_{\text{max}}$. Thus, from:

$$c \leq c_{\text{max}},$$

results

$$H_{\text{min}} = \frac{m}{c_{\text{max}}} S.$$  

(10)

Usually (9) or (10) are imposed only for $i = 1$, that is on the first pipe – in the case of continuous levigation, or on the first step – in the case of intermittent levigation, since with every increase of a granulometric class disappears from the continuous phase and thus the solid concentration in the suspension decreases.
Decanting process duration for continuous levigation

For continuous levigation, decanting of a granular material into \( N \) granulometric classes is considered finalized when all particles belonging to each class \( i = (1, N - 1) \) reach by sedimentation the bottom of the corresponding selection pipe.

The whole quantity of granular material is initially introduced in the first pipe \( (i = 1) \) through the upper extremity of the pipe. Decanting in this first pipe is finished when all particles with diameter \( d \geq d_i \) travel the height \( H_i \) by sliding through the liquid, which takes a time \( t_i \), given by:

\[
t_i = \frac{H_i}{v_a(d_i)} \quad \text{(11)}
\]

which in the limit tends to infinity since \( v_a(d_i) = 0 \) (8). This observation is valid for the other pipes as well \((i > 1)\) for which the duration of decanting \( t_i \) is given by the time particles with diameter \( d \leq d_i \) need to travel the height \( H_i \) of pipe \( i \) and pass through to the next pipe \( i+1 \):

\[
t_i = \frac{H_i}{v_a(d_i)} \quad \text{(12)}
\]

where, again, \( v_a(d) = 0 \).

As a result, finite and reasonable values for \( t_i \) \((i = (1, N - 1))\) are obtained by limiting the duration of backwash sedimentation in each pipe to the time needed by most particles in each granulometric class to slide to the pipe's basis (for the pipe with \( i = 1 \)) or pass through to the next pipe \((i > 1)\), respectively. For example, the fraction of the particles involved in the above relation can be defined as \( d \geq 1.05 d_i \) for the first pipe, and \( d \leq d_i \) for the other pipes, or in general, \( d \geq b d_i \) and \( d \leq d_i/b \) where \( b > 1 \).

Denote \( d_i^* = b d_i \) and \( d_i'' = d_i/b \); then the corresponding values for \( t_i \) \((denoted by \( t_i'\))\) are obtained by specializing (11), (12):

\[
t_i' = \frac{H_i}{v_a(d_i')} \quad \text{(13)}
\]

for \( i = (1, N - 1) \).

Thus, the total duration for the decanting of the \( N \) granulometric classes for \( d_i^* = d_i'' \), \( i = (1, N - 1) \) is:

\[
i' = \max \{t_1, t_2, ..., t_i', ..., t_{N-2}\} \quad \text{(14)}
\]

After this time \( t' \), each pipe will contain particles – of diameter \( d_i \) – which did not move at all \((v_a(d_i) = 0)\) and particles with diameter in the range \( (d_i, d_i'')\) which moved little and are still in suspension. Thus, particles with diameter \( d_i \) in the first pipe have null speed \((v_a(d_i) = 0)\) and are still located in the upper part of this pipe, while particles with diameter \( d \in (d_i, d_i'') \) are descending to the pipe's basis. In all other pipes \((i > 1)\) particles of diameter \( d_i \) and speed \( v(d) = 0 \) are located at the bottom of the pipe \( i \), while particles of diameter \( d \in (d_i, d_i'') \) are spread along the height \( H_i \) of pipe \( i \).

In order to also have these remaining particles travel the height of their corresponding pipe to the bottom of their pipe, the pumps are stopped \((Q = 0)\), which results in the transport speed of the liquid to become null \((v_{wi} = 0)\) while the absolute speed of the ensemble of these particles becomes strictly negative \((v = -v\) from (3)). The sedimentation time of these particles which are in suspension when stopping the pumps is given by \( t_{i''} \):

\[
t_{i''} = \frac{H_i}{v_a(d_i'')} \quad \text{(15)}
\]

In this second step of the decantation process corresponding to the installation being stalled without pumping \((Q = 0)\), the ascending speed of the liquid in each pipe is null \((v_{wi} = 0)\) and thus \( v_a(d) = -v(d) = -K d_i^2 < 0; \) further:

\[
t_{i''} = \frac{H_i}{Kd_i} \quad \text{(16)}
\]

The necessary time for this second step to be finalized in all the levigation installation pipes is:

\[
t' = \max \{t_1', t_2', ..., t_N'\} \quad \text{(17)}
\]

Finally, the total duration \( t_i \) needed for sedimentation of all particles in each pipe is:

\[
t_i = t_i' + t_{i''}, \quad i = [1, N - 1] \quad \text{(18)}
\]

We can now introduce an additional criterion for setting the heights \( H_i \), namely the condition that each of the two steps of the separation process (the first one with \( Q > 0 \), and the second one with \( Q = 0 \)) have the same duration for each of the installation pipes:

\[
t_1 = t_2 = ... = t_i = ... = t_{N-1} = t' \quad \text{(19)}
\]

and

\[
t_1 = t_2 = ... = t_i = ... = t_{N-1} = t'' \quad \text{(20)}
\]

These conditions result in:

\[
\frac{H_i}{v_a(d_i')} = \frac{H_i}{v_a(d_i'')} \quad \text{(21)}
\]

and respectively:

\[
\frac{H_i}{Kd_i'} = \frac{H_i}{Kd_i''} \quad \text{(22)}
\]

where \( i = (1, N - 2) \).

From ((21) and (22)) results the recurrence:

\[
\frac{H_i}{H_{i+1}} = \left(\frac{d_{i+1}}{d_i}\right)^2 \quad \text{or} \quad H_{i+1} = H_i \left(\frac{d_{i+1}}{d_i}\right)^2 \quad \text{(23)}
\]

In order to compute \( t' \) and \( t'' \), the height of one of the \( N \) pipes is fixed such that:

\[
H_{i+1} = \frac{m}{v_{wi}(d_i)} \quad \text{(24)}
\]

while the heights of the other pipes are computed using (23).

If, for example, \( H_i \) is the first value to be fixed in the \( H \) sequence, then the total duration of the levigation process is:

\[
t = t_1' + t' = \frac{H_1}{v_a(d_1')} + \frac{H_1}{Kd_1'} = \frac{b^3}{b^2 - 1} \frac{H_1}{Kd_1'} \quad \text{(25)}
\]

Denote by \( b \) the fraction (by diameter) of the solids in suspension when stopping the pumps (when the value of the flow \( Q \) instantaneously decreases to zero).
Decanting process duration for intermittent levigation

For intermittent levigation the total duration \( t \) of the decanting process equals the sum of the decanting durations for the first \( N-1 \) granulometric classes:

\[
t = \sum_{i=1}^{N-1} t_i = \sum_{i=1}^{N-1} [t'_i + t''_i].
\]  

(26)

The values of \( t'_i, t''_i \) have the same significance as for continuous levigation and are computed similarly (using (13) and (16)).

Experimental part

For the experimental part, we used a granular material previously sorted into \( N = 4 \) granulometric classes, delimited by the following values of the diameter \( d_i \): \( d_1 = 80 \mu m, d_2 = 60 \mu m \) and \( d_3 = 40 \mu m \). The amount \( m' \) from each class had been chosen so that the total amount \( m' = \sum m_i \) was 10 grams (table 1). The resulting total amount was mixed and then decanted again using an installation designed using the algorithm in this paper.

The decanting experiment was performed after the design and manufacturing of the installation, using as process parameters the values obtained by the proposed algorithm. Following decanting of the material probe using the pilot installation the amount of each granulometric class was again weighed and compared to the value prior to decanting. The validity of our algorithm was tested by comparing these values before and after the decanting process: for each granulometric class the weights before levigation \( (m'_i) \) and after levigation \( (m''_i) \) were obtained.

The initial data (the parameters and their values obtained or pre-determined before the experiment design) used in the algorithm were as follow: water density \( \rho_w = 1000 \text{ kg/m}^3 \), dynamic viscosity of water \( \eta_w = 1.01 \times 10^{-3} \text{ Ns/m}^2 \), solid material density \( \rho_s = 2500 \text{ kg/m}^3 \), mass of the material probe \( m = 10 \text{ g} \), and maximum concentration of solids dispersed in the liquid environment \( c_{max} = 20 \text{ g/dm}^3 \). The sequence of numerical calculations for the computation of the installation dimensions and the working parameters as given by our algorithm was as follows:

- Compute the minimum value of the diameter \( D_i \) for the first selection pipe using \( D_{\text{min}} = 100 d_1 \). We obtain \( D_{\text{min}} = 8 \text{ mm} \) and choose as diameter for the first selection pipe \( D_1 = 25 \text{ mm} \).
- From the recurrence relation (6) we obtain: \( D_2 = 33.4 \text{ mm} \) and \( D_3 = 50 \text{ mm} \).
- Compute the minimum values for \( H_i \) in (10): we obtain \( H_{\text{min},1} = 1018 \text{ mm}, H_{\text{min},2} = 570 \text{ mm}, \) and \( H_{\text{min},3} = 254 \text{ mm} \). Choose the value for the first pipe as \( H_1 = 1020 \text{ mm} \). The subsequent values are obtained using (23); we obtain \( H_2 = 574 \text{ mm} \) and \( H_3 = 255 \text{ mm} \).
- The necessary flow for levigation (for the pumping step) is computed using (7) resulting in \( Q = 2.57 \text{ cm}^3/\text{s} \).
- The total duration \( t \) of the levigation process as well as the duration of the zero flow levigation step \( t'' \) is computed using (25) and (16) respectively, and the duration of the pumping step \( t' \) is given by the difference \( t = t' + t'' \); take \( b = 1.25 \). We obtain \( t = 9 \text{ min} 2s, t'' = 3 \text{ min} 15 \text{ s} \) and \( t' = 5 \text{ min} 47s \).

Experiment description

A pilot installation was designed and constructed in the specialized laboratories of Universitatea Petrol - Gaze Ploiești; this allowed the comparison of the theoretical predictions with the experimental results. The continuous levigation installation was first manufactured using three selection pipes with diameters \( D_i \) and heights \( H_i \) (\( i = 1, 2, 3 \)) obtained from numerical calculations using our algorithm: \( \{25, 33.4, 50\} \) mm and \( \{1020, 574, 255\} \) mm respectively (fig. 4). The three selection pipes were connected in series using connecting pipes. Water was used as both dispersion and circulation liquid.

Results and discussions

The values measured before \( (m'_i) \) and after \( (m''_i) \) levigation for the mass \( m_i \) of each of the four granulometric class are listed in Table 1 (one table row for each probe and each decanting test). For all five continuous levigation tests in the experiment we used the same levigation installation described above and the same values for parameters \( Q, t, t' \). The difference in the tests stands in the granulometric composition of probes (the values \( m'_i \)) and, obviously, in the final results after levigation (the values \( m''_i \)).

The differences between the values measured before and after the experiment were computed (in percentage change of the final value with respect to the initial value), for each granulometric class and for the whole material sample, as follows:

\[
e_i = \frac{(m''_i - m'_i)}{100 m'_i} \times 100
\]  

(27)
and, respectively,

$$\varepsilon = (m'' - m') \cdot 100/m'$$  \hspace{1cm} (28)$$

Note that the absolute values of these changes are below 1%, and in general $m'' < m'$. Changes between values measured before and after levigation may also be due to possible material sample losses during the experiment.

Conclusions
A main novelty of our algorithm is the two step decanting procedure (a pumping period followed by a 'wait' period when the pumping is stopped) which results in a decrease in the time duration needed for sedimentation.

Our condition which imposes that the pumping and waiting period (and thus also the decanting period for each granulometric class) have equal durations is novel as well.

Under the above constraints, we obtain recurrence relationships between the selection pipes diameters, heights and limit diameters among granulometric classes (5), (23).

Experimental verification of our algorithm resulted in <1% relative differences with respect to the theoretical predictions.

We also conclude that the use of our algorithm decreases the decanting process duration. Finally, when used on an industrial scale, our algorithm may decrease production costs by an increase in productivity and a reduction in the energy consumption needed for pump operation.

References
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