Dynamic Parameters Optimization for the Vibrating Sieve with Two Granular Material Sizing Units, Working in Resonance

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This paper concerns the dynamics of a vibrating sieve for granular material, working in resonance, composed of two sizing units and driven by using an elastic controlling-rod. Based on specific hypothesis and technological requirements for the sieving of granular material, the computational relations for the main functional and constructive parameters of the vibrating sieve were established. Based on these theoretical considerations, a new vibrating sieve, working in resonance, has been proposed for granular material sizing.

Keywords: granular material, vibrating sieve, dynamic parameters, technological vibrations, power consumption

In order to increase the technological and economical efficiency when treating the granular materials in processing industries, modern and advanced equipment is required. The performance of such equipment is characterized by global quality parameters, so that to obtain technological productivity, with low energy consumption [1,2]. Such technological requirements for sizing of granular material have motivated the work presented in this paper, i.e., the design of a vibrating sieve working in resonance [2-5]. The sieves may also be used for various studies on granular material, e.g., the grain-size distribution of a disperse product [6].

Constructive and functional technical solution

The technical solution proposed for the vibrating sieve working in resonance is presented in figure 1. Mainly, this equipment is composed of the following elements: upper frame 1, lower frame 2, balance beam (balancer) 3, elastic elements attached to the controlling-rod 4, driving shaft 5, elastic elements coupling (between the two sizing units) 6, support 7. In order to obtain an adequate technical solution, the following constructive and functional conditions were considered:

- the vibrating regime must lead to a good quality of the granular material sieving;
- for the same sieving capacity as when using classical sieves, the metal quantity needed to build the newly proposed sieve must be reduced with about 40%;
- for the same sieving capacity as when using classical sieves, the energy consumption of the new sieve must be reduced with about 75% [3,7].

The sieving operation is characterized by unidirectional vibrating displacement of the upper frame 1 and of the lower frame 2. These unidirectional vibrations are induced by the driving shaft system 5 and by the elastic elements attached to the controlling-rod 4. As for the balance beams 3, they are attached to the support 7. The entire mass of the sieve and granular material is supported by this balance beams and support system. The two sizing units are mounted on both upper and lower frames [3,8].

Dynamic model

The technical solution shown in figure 1 was schematized by the dynamic model given by figure 2. The sizing units have been modelled by the masses \( m \) and \( m_s \); the viscoelastic elements connecting the two sizing units are characterised by the complex stiffness coefficient \( k \), while the viscoelastic elements attached to the controlling-rod are characterised by the complex stiffness coefficient \( k_o \) [7,9,10].

Since the ratio between the vibrations amplitude and the length of the driving shaft is negligible, one can consider the hypothesis that the sizing units have just translational motion following the direction of the starting angle. Thus, the motion is characterised only by unidirectional vibrations. The eccentric mechanism of the controlling-rod is fixed on a sizing unit, while the controlling-rod extremity is fixed on the other sizing unit. This means that the mechanical system is internally disturbed. In what concerns the global center of mass of the mechanical system, it remains fixed if the sizing units and their supporting frame are inertially and geometrically identical and if the granular material is uniformly distributed on the sizing surface.

Otherwise, either because of some local agglomeration of granular material or because of inertial/geometrical disproportions between the two sizing units, the center of mass changes its position, which introduces internal linking forces in the joints of the balance beams.

Functional parameters determination

Depending on the type of granular material to be sieved and on the hourly sieving capacity, one can establish the sieving performance level as a function of the following parameters: the amplitude of the sizing units vibrations; the eccentricity of the controlling-rod extremity; the amplitude variation of the controlling-rod extremity; the number of functions of sieving capacity.
the angular frequency of the forced vibrations; the force transmitted at the elastic controlling-rod; the driving power and the cranking torque of the sieve equipment.

Fig. 2. Dynamic simulation model: \(x_1\) – unidirectional displacement of the upper sizing unit, \(x_2\) – unidirectional displacement of the lower sizing unit, \(x_0\) – unidirectional displacement of the controlling-rod extremity which contains the eccentric controlling-rod bushing; \(G_1^*\) – complex shear modulus of the viscoelastic link between the two sizing units; \(G_0^*\) – complex shear modulus of the viscoelastic elements attached to the controlling-rod; \(\omega\) – the disturbing angular frequency, i.e., the angular frequency of the driving shaft.

Differential equations of motion

For the dynamic simulation model shown in figure 2, it corresponds the following system of differential equations of motion:

\[
\begin{align*}
    m_1\ddot{x}_1 + (k_0 + k_1)(x_1 - x_2) &= k_0 x_0, \\
    m_2\ddot{x}_2 - (k_0 + k_1)(x_1 - x_2) &= -k_0 x_0,
\end{align*}
\]

(1)

where \(k_0\) is equivalent stiffness coefficient of the elastic elements attached to the controlling-rod and \(k_1\) is equivalent stiffness coefficient of the elastic elements connecting the two sizing units.

By adding the two equations of system (1), one obtains:

\[
    m_1\ddot{x}_1 + m_2\ddot{x}_2 = 0,
\]

(2)

or

\[
    \frac{m_1}{m_2} = -\frac{\ddot{x}_1}{\ddot{x}_2} = \mu.
\]

(3)

The negative sign in (3) means that the sizing units are out of phase, more precisely:

- If the elements attached to the controlling-rod and those connecting the two sizing units are purely elastic, then the phase shift \(\varphi\) between the motions of the two sizing units is \(\varphi = \pi\);
- If the elements attached to the controlling-rod and those connecting the two sizing units are characterized by viscoelastic properties, then the phase shift \(\varphi\) between the motions of the two sizing units is in the range \((0, \pi)\).

Depending on the ratio \(\mu = m_1 / m_2\), one can distinguish two cases:

- If \(\mu = 1\), i.e., \(m_1 = m_2\), then the mechanical system is well-balanced, so there is no dynamic force transmitted outside;
- If \(\mu \neq 1\), i.e., \(m_1 \neq m_2\), then the system is unbalanced, in this case it transmits dynamic force outside, which means power loss.

Let us denote by \(x = x_1 - x_2\) the relative displacement, on the direction of the working vibrations, between the two masses \(m_1\) and \(m_2\). The differential equations of motion (1) can be also written as:

\[
\begin{align*}
    \ddot{x}_1 + \left(\frac{k_0 + k_1}{m_1}\right)(x_1 - x_2) &= \frac{k_0}{m_1} x_0, \\
    \ddot{x}_2 - \left(\frac{k_0 + k_1}{m_2}\right)(x_1 - x_2) &= -\frac{k_0}{m_2} x_0.
\end{align*}
\]

By deducing the second equation from the first equation above, it comes:

\[
    \ddot{x}_1 - \ddot{x}_2 + \left(\frac{k_0 + k_1}{m_1} + \frac{m_1 + m_2}{m_1 m_2}\right)(x_1 - x_2) = k_0 x_0 \frac{m_1 + m_2}{m_1 m_2},
\]

(4)

or

\[
    \ddot{x} + \left(\frac{k_0 + k_1}{m_2}\right) x = \frac{k_0}{m_2} x_0 \frac{1}{m_2},
\]

(5)

where \(\ddot{x} = \ddot{x}_1 - \ddot{x}_2\) denotes the relative acceleration between the two sizing units and \(m = \frac{m_1 m_2}{m_1 + m_2}\) is the reduced mass of the system.

Because of using only rubber elements in the construction of the sieve equipment, then the equivalent stiffness coefficient of an elastic-elastic system is given by:

\[
    k = \lambda G^*\]

(6)

where \(G^* = G(1 + j\delta)\) is the complex shear modulus and \(\lambda\) denotes the geometric multiplying coefficient.

The complex shear modulus describes the rubber elastic behaviour by \(\text{Re}(G^*) = G\) and the viscous behaviour (internal energy dissipation) by \(\text{Im}(G^*) = G\delta\), where \(G\) is the shear modulus and \(\delta\) is the internal loss angle [9,10].

For parallelepipedic rubber elements, subjected either to compression or to lateral slip (shear), the geometric multiplying coefficient is given by:

\[
    \lambda = (1 + \beta \Phi^2) \frac{S}{h} \text{ for compression.}
\]

(7)

\[
    \lambda = \frac{S}{h} \text{ for shear.}
\]

(8)

In relations (7) and (8), the parameters denote:

- \(S\) – total equivalent working area of the rubber antivibration elements;
- \(h\) – height (thickness) of a rubber antivibration element;
- \(\Phi\) – shape coefficient of the rubber element, which is the ratio between loaded at compression area and free area;
- \(\beta\) – multiplying factor taking into account the composition of the rubber compound.

By introducing relation (6) into (5), one obtains the following complex differential equation:

\[
    \ddot{x} + \left(\frac{1}{m} \left(\lambda_0 G_1^* + \lambda_1 G_1^*\right)\right) \ddot{x} = \frac{1}{m} \lambda_0 G_0^* \ddot{x}_0,
\]

(9)

with

\[
    \ddot{x}_0 = r e^{im\omega},
\]

(10)

\[
    \ddot{x} = \bar{A}^* e^{im\omega},
\]

(11)

where \(r\) is the eccentricity of the controlling-rod bushing (extremity) and \(\bar{A}^*\) denotes the amplitude of the relative displacement between the two sizing units, expressed in complex form.

The resolution of the differential equation (9) provides the expression of the complex amplitude \(\bar{A}^*\):
\[ A^* = \frac{C}{p^2 + Q^2} \left[(P + Q\delta_0) + j(P\delta_0 - Q)\right], \]  

(12)

where the following notations were used:

\[ P = -m\omega^2 + \lambda_0 G_0 + \lambda_1 G_1, \]
\[ Q = \lambda_0 G_0 \delta_0 + \lambda_1 G_1 \gamma, \]
\[ C = \tau \lambda_0 G_0. \]

Let us now consider the definition of the complex measure \( A^* \):

\[ A^* = A (\cos \varphi + j \sin \varphi) = A e^{j\varphi}, \]  

(13)

where \( A \) is the amplitude of the relative displacement between the two sizing units, on the direction of the working vibrations, and \( \varphi \) is the phase shift between the motions of the two sizing units.

By identification between (12) and (13), one obtains the expressions of \( A \) and \( \varphi \), as follows:

\[ A = r \lambda_0 G_0 \left[ 1 + \delta_0 \right] \]
\[ \left( -m \omega^2 + \lambda_0 G_0 + \lambda_1 G_1 \right)^{1/2}, \]  

(14)

\[ \arg \varphi = \left( -m \omega^2 + \lambda_0 G_0 + \lambda_1 G_1 \right) \delta_0 + \left( \lambda_0 G_0 \delta_0 + \lambda_1 G_1 \right) \delta_1. \]  

(15)

The relative displacement \( x \) is defined as:

\[ x = \text{Im} \bar{x} = A \sin(\omega t + \varphi), \]  

(16)

while the displacements and the amplitudes of the two sizing units are given by:

\[ x_1 = \frac{1}{1 + \mu} x; \quad x_2 = \frac{-\mu}{1 + \mu} x, \]  

(17)

\[ A_1 = \frac{1}{1 + \mu} A; \quad A_2 = \frac{-\mu}{1 + \mu} A. \]  

(18)

**Elastic controlling-rod force**

The actuating force provided by controlling-rod is equal to the deformation force of the elastic elements attached to the controlling-rod, with a negative sign:

\[ F = -\frac{\partial \Pi_0}{\partial \delta_0}, \]  

(19)

where \( \Pi_0 = \frac{1}{2} \lambda_0 G_0 (x_1 - x_2 - x_0)^2 \) is the deformation potential energy of the elastic elements attached to the controlling-rod [8].

Based on relation (19), the complex expression of the controlling-rod force is:

\[ \bar{F} = k_0 \bar{x} - k_0 \bar{x}_0, \]

\[ \bar{F} = k_0 G_0 (\bar{x} - \bar{x}_0), \]  

(20)

where \( \bar{F} = F^* e^{j\theta} \) with \( \theta \) denoting the phase shift between the controlling-rod force and the relative displacement, and where \( \bar{x}_0 = re^{j\theta} \) (relation (10)), with \( r \) already defined as the eccentricity of the controlling-rod bushing.

From the complex relation (20) and taking into account (10) and (11), it comes:

\[ F^* = \lambda_0 G_0^* (A^* - r), \]  

(21)

By replacing the complex expressions of \( G^* \) and \( A^* \) in (21), one obtains:

\[ F^* = \lambda_0 G_0^* \left[ (A \cos \varphi - A \delta_0 \sin \varphi - r) + j (A \sin \varphi - A \delta_0 \cos \varphi - r \delta_1) \right] \]  

(22)

Finally, one obtains the expression of the controlling-rod force:

\[ F = \text{Im} (\bar{F}) = F_0 \sin(\omega t + \theta), \]  

(23)

with \( F_0 \) and \( \theta \) given by:

\[ F_0 = \lambda_0 G_0 \left[ (1 + \delta_0^2) (A^2 + r^2 - 2Ar \cos \varphi) \right], \]  

(24)

\[ \arg \theta = \frac{A \sin \varphi + \delta_1 A \cos \varphi - \delta_0 r}{A \cos \varphi - \delta_1 \sin \varphi - r}. \]  

(25)

If \( \delta_0 = \delta_1 = 0 \), then:

\[ F_0 = \lambda_0 G_0 R, \]  

(26)

where:

\[ R = (A^2 + r^2 - 2Ar \cos \varphi)^{1/2} = |r - A|, \]  

(27)

\[ A = r \lambda_0 G_0 \left| \frac{1}{-m \omega^2 + \lambda_0 G_0 + \lambda_1 G_1} \right|. \]  

(28)

In this case, from (27) and (28) it results the following expression of the function \( R = R(\omega) \):

\[ R(\omega) = r \left| 1 - \frac{\lambda_0 G_0}{-m \omega^2 + \lambda_0 G_0 + \lambda_1 G_1} \right|. \]  

(29)

This function \( R = R(\omega) \) is plotted in figure 3. It can be observed that function \( R(\omega) \) and implicitly the controlling-rod force function \( F_0 \) are null for two distinguished values of the angular frequency, namely:

\[ \omega = p_0 = \sqrt{\frac{\lambda_0 G_0}{m}}, \]  

(30)

\[ \omega = p_0 = \sqrt{\frac{2 \lambda_0 G_0 + \lambda_1 G_1}{m}}, \]  

(31)

where \( p_0 \) denotes the angular frequency of the system in pre-resonance regime, while \( p_1 \) is the angular frequency of the system in post-resonance regime.

Function \( R(\omega) \) and implicitly the force \( F_0 \) achieve their maximum value for the following angular frequency:

\[ \omega = \rho = \sqrt{\frac{\lambda_0 G_0 + \lambda_1 G_1}{m}}, \]  

(32)

where \( \rho \) denotes the natural angular frequency.

**Vibrating sieve Driving power**

The driving power has two main components, i.e.: power \( N_1 \) for overcoming the friction in sieve bearings and power \( N_2 \) for maintaining the vibration regime imposed by the sieving process.

The maximum power needed to overcome the friction forces in the driving shaft bearings is:

\[ N_1 = z N_1, \]  

(33)
where \( z \) denotes the number of identical bearing pairs and \( N'_{\text{fr}} \) is the power needed to overcome the friction corresponding to a pair of identical bearings, given by:

\[
N'_{\text{fr}} = 0.5 f_\text{fr} F_0 \omega d_r. \tag{34}
\]

In (34), \( f_\text{fr} \) denotes the coefficient of friction in bearings, \( d_r \) is the equivalent diameter of the cylindrical friction surface and \( \omega \) is the angular frequency of the technological vibrations.

The mechanical work performed by the controlling-rod force in order to maintain the technological vibrations regime, over a period \( T \), is:

\[
L = \int_0^T F \, \dot{x}_o \, dt = \int_0^T F_0 \, \dot{x}_o \, \sin(\omega t + \phi) \, dt. \tag{35}
\]

Taking into account that \( \dot{x}_o = \omega \cos(\omega t) \) (obtained by differentiation of (10)), one can easily obtain:

\[
L = \pi F_0 r \sin \theta. \tag{36}
\]

The average mechanical power needed to maintain the steady-state vibrations of the system, over a full period \( T \), is:

\[
N_1 = \frac{L}{T} = 0.5 r F_0 \omega \sin \theta. \tag{37}
\]

Finally, the total power required at the driving shaft of the vibrating sieve is given by:

\[
N = N_1 + N_2 = 0.5 F_0 \omega \left( z \, d_r, f_r + r \, \sin \theta \right) \tag{38}
\]

and the total power required at the driving motor is:

\[
N_m = \frac{N}{\eta} = \frac{0.5 F_0 \omega \left( z \, d_r, f_r + r \, \sin \theta \right)}{\eta}, \tag{39}
\]

where \( \eta \) is the efficiency of the power transmission from the motor to the sieve driving shaft.

The cranking torque

The cranking torque \( M_\text{pr} \) is the torque which must be provided by the driving motor at the starting time:

\[
M_\text{pr} = F_r \cos \omega t, \tag{40}
\]

where \( F_r = \lambda_0 G (x - x) \) is the required starting force.

The relative displacement \( x \) at the starting time results from the equation of equilibrium of the elastic forces involved in the mechanical system, more precisely:

\[
x = \frac{r \lambda_0 G_0}{\lambda_0 G + \lambda_1 G_1} \sin \omega t. \tag{41}
\]

Introducing relation (41) in the expression of the starting force \( F_\text{p} \) and then in (40), one finally obtains:

\[
M_\text{pr} = 0.5 r^2 \frac{\lambda_0 \lambda_1 G_0 G_1}{\lambda_0 G_0 + \lambda_1 G_1} \sin 2 \omega t, \tag{42}
\]

with the following maximum value of the cranking torque:

\[
M_{\text{pr}}^{\text{max}} = 0.5 r^2 \frac{\lambda_0 \lambda_1 G_0 G_1}{\lambda_0 G_0 + \lambda_1 G_1}. \tag{43}
\]

The driving motor succeeds to start the sieving equipment if:

\[
\frac{M_{\text{regime \ needed}}}{M_{\text{pr}}^{\text{max}}} < \frac{M_{\text{motor}}}{M_{\text{pr}}^{\text{max}}}, \tag{44}
\]

where \( M_{\text{regime \ needed}} = 9550 \, N \cdot m / \text{nom} \) is the torque of the driving motor during the working regime, with \( N \) [kW] the power of the motor and \( \omega \) [rot/min] the normal speed of the motor. The ratio \( M_{\text{regime \ needed}} / M_{\text{motor}} \) represents the starting characteristic of the driving motor considered.

Dynamic regime parameters

The main criteria when choosing the dynamic regime parameters of the vibrating sieve are:

- achievement of the required technological vibrations level, by a good correlation between the amplitude and, respectively, the frequency values;
- disturbance of the vibrating sieve for minimum values of the elastic controlling-rod force;
- achievement of the normal vibrating regime for minimum driving power consumption.

As an example, let us present below the dynamic regime parameters of a experimentally validated vibrating sieve, working in post-resonance, with the following technical characteristics:

- sieving (sizing) surface = 2 x 6 m²;
- total vibrating mass = 1744 kg;
- starting angle of the directed vibrations, with respect to the horizontal direction = 40°;
- angular frequency of the technological vibrations = 75.4 rad/s;
- amplitude of the relative displacement between the two sizing units, on the direction of the working vibrations = 5.5 mm;
- power of the driving motor = 3.0 kW;
- eccentricity of the controlling-rod bushing (extremity) = 7 mm.

Based on the considered constructive and functional hypothesis and using the calculation formula previously developed in this paper, the following dynamic regime parameters were established:

- mass of the upper frame: \( m_1 = 910 \) kg;
- mass of the lower frame: \( m_2 = 834 \) kg;
- reduced mass of the system: \( m = 435 \) kg;
- equivalent stiffness coefficient of the elastic elements attached to the controlling-rod: \( k_0 = 0.56 \times 10^6 \) N/m;
- equivalent stiffness coefficient of the elastic elements coupling between the two sizing units: \( k_1 = 1.24 \times 10^6 \) N/m;
- natural angular frequency (at resonance) = \( \omega_r = 64.3 \) rad/s;
- pre-resonance angular frequency: \( \omega_p = 53.4 \) rad/s;
- rubber elements internal loss angle: \( \phi = 0.15 \);
- eccentricity of the controlling-rod bushing: \( r = 7 \) mm;
- amplitude of the relative displacement between the upper and lower sizing units: \( \Delta = 5.46 \) mm;
- elastic controlling-rod force: \( F_0 = 2000 \) N;
- total power required at the driving shaft, for steady-state regime: \( N = 550 \) W; 
- peak power (required to attend post-resonance): \( N' = 2000 \) W.

The evolutions of some dynamic regime parameters were plotted in figures 3, 4 and 5, as a function of the angular frequency of the forced vibrations. Thus, one must choose from figure 3 the values \((\Delta, \omega)\) of the amplitude and the angular frequency, such as that they correspond to a minimum value of the elastic controlling-rod force in
Fig. 3. The amplitude of the relative displacement between the upper and lower sizing units, as a function of the angular frequency of the technological vibrations.

Fig. 4. The elastic controlling-rod force as a function of the angular frequency of the technological vibrations.

Fig. 5. The power required at the driving shaft as a function of the angular frequency of the technological vibrations.

Conclusions
The technical and economical advantages of the vibrating sieves working in resonance recommend them for more intensive use in the technological flow sheets of intensive energy consuming processing industries.

Compared with the classical sieves, for the same granular material sieving capacity, the vibrating sieves working in resonance require low energy consumption, low metal consumption, while working with reduced dynamic forces. This sieve equipment working in resonance involves new design and constructive issues.

The constructive and functional technical solution and the theoretical calculations presented in this paper were accomplished by the design, realization and testing of the CR 2 x 7.5 type of vibrating sieve working in resonance.

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