Analysis of Technological and Functional Parameters of Oscillating Mills for Granular Material Grinding

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This paper concerns the dynamics of oscillating grinding mills supported on rubber vibration isolation elements. Computational relations are provided for the functional and technological parameters of the system, as well as for the parameters characterizing the vibration isolation subsystem.

Keywords: granular material, grinding mill, vibration isolation, rubber elements

The oscillating mill with grinding bodies (grinding balls or rods) is a technological equipment specialized in very fine grinding of granular materials. These oscillating grinding mills and the other types of grinding mills are used in the technological flow sheets of processing industries, in order to obtain very fine ferroalloy powders (5-10 μm), or small size scrap or crushed marble [1-4].

This paper presents the technical solution and the computation relations of oscillating mills with two grinding chambers (compartments), excited by inertial vibration shakers (generators) with rotating forces [5].

Based on computational relations issued from both dynamical and experimental studies performed on a Palla-U35 oscillating grinding mill, manufactured by Humboldt company (Germany), it was possible to improve the existing oscillating mills for granular material and to realize new oscillating grinding mills.

The oscillating grinding mill is composed of the following main elements (fig. 1):

- the grinding chambers (compartments) 1,2, identical, located symmetrically with respect to C, longitudinal axis, where C is the mass center of the system;
- inertial vibration generator 3, producing a rotating force with the axis passing by O and parallel to C axis, where O is the center of the disturbing (perturbation) force;
- vibration isolation support system 4, made of rubber elements;

The dynamic analysis of oscillating grinding mills must solve the following two main problems [1,3,5]:
- to establish the dynamic regime corresponding to the desired technological vibrations;
- to determine the support system solution which ensures the desired vibration isolation level.

Functional parameters determination

Figure 2 shows the dynamic simulation model of the oscillating mill with two grinding chambers, taking into account its constructive and technological particularities.

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The oscillating grinding mill operates in post-resonance for all degrees of freedom: \( \omega = (3\ldots10)\rho_j \), where \( \omega \) is the angular frequency of the disturbing force, and \( \rho_j \) is the natural angular frequency corresponding to eigenmode \( j \) [6].

In this case, since the influence of the dissipative forces is insignificant, these dissipative forces can be neglected in the simulation model. In what concerns the disturbing force, we have considered a centrifugal force of inertia, oriented such that its direction should not pass through the mass center of the system. Also, we have considered the case in which the mass center of the system does not belong to the horizontal plane of the vibration isolation elements. With all these assumptions, the oscillating grinding mill is modeled as a rigid body with two vertical planes of symmetry, supported by identical vibration isolation elements, and with the disturbing force application point different from the mass center [5,7].

**Differential equations of motion**

The system has three degrees of freedom, being excited following the translation directions \( C_x \) and \( C_z \) and following the rotation direction \( \varphi \). Let us denote by \( X \) the coordinate following \( C_x \) direction, by \( Z \) the coordinate following \( C_z \) direction, and by \( \varphi \) the rotation coordinate [5,8,9].

For the elastic system modeled in figure 2, the system of differential equations of motion can be written as [5,9,10]:

\[
\begin{align*}
\ddot{X} + 4k_x X + 4a_x k_x \varphi &= P_x \cos \omega t, \\
J_x \ddot{\varphi} + 4\varphi_x (k_x a_x^2 + k_x) + 4k_x a_x X &= z_x P_x \cos \omega t, \\
m \ddot{Z} + 4k_z Z &= P_z \sin \omega t,
\end{align*}
\]

where:

- \( m \) is the total sprung mass of the grinding mill;
- \( J_x \) – moment of inertia of the grinding mill with respect to \( C_x \) axis;
- \( k_x \) – stiffness coefficient of an elastic element on \( C_x \) direction;
- \( a_x \) – horizontal distance between the longitudinal median plane and the elastic supports;
- \( a_z \) – vertical distance between the horizontal plane containing the elastic elements and the position of the mass center;
- \( z_x \) – vertical distance, measured in the longitudinal median plane, between the mass center \( C \) and the vibration center \( O \);
- \( P_x \) – amplitude of the disturbing centrifugal force of inertia, with \( P_x = m \omega^2 \rho_x \);
- \( \omega \) – angular frequency of the disturbing force;
- \( m_z \) – total eccentric mass with respect to the rotation axis of the unbalanced elements of the grinding mill;
- \( r \) – distance between the rotation axis and the mass center of the unbalanced elements of the grinding mill;
- \( m_f \) – static moment of the unbalanced elements of the grinding mill.

For the eigenmode corresponding to decoupled translation vertical vibrations, the natural angular frequencies of the system are given by:

\[
\omega_j^2 = \frac{k_x}{m},
\]

As for the eigenmode corresponding to the \( (X, \varphi) \) coupling between translation \( C \) vibrations and rotation \( \varphi \), vibrations, the natural angular frequencies of the system are as follows:

\[
\frac{p_j^2}{\rho_j^2} = \frac{1}{2} \left[ \frac{k_z}{k_x} \left( \frac{a_z^2}{\rho_z^2} + \frac{a_z^2}{\rho_z^2} + \frac{a_z^2}{\rho_z^2} - \frac{4k_z}{k_x} \left( \frac{a_z}{\rho_z} \right)^2 \right) \right]^2.
\]

where \( \rho_x \) stands for the radius of gyration, defined by \( J_x = m \rho_x^2 \).

In what concerns the regime of forced vibrations, the solutions of the differential equations (1) are harmonic, with the form suggested by the right term of (1). The displacement amplitudes are given by:

\[
\begin{align*}
A_x &= m \rho_x \omega^2 \frac{1}{4k_x} \left( \frac{2k_x}{k_x} \left( \frac{a_z^2}{\rho_z^2} + \frac{a_z^2}{\rho_z^2} + \frac{a_z^2}{\rho_z^2} - \frac{4k_z}{k_x} \left( \frac{a_z}{\rho_z} \right)^2 \right) \right), \\
A_z &= m \rho_x \omega^2 \frac{1}{4k_z} \left( \frac{2k_z}{k_z} \left( \frac{a_z^2}{\rho_z^2} + \frac{a_z^2}{\rho_z^2} + \frac{a_z^2}{\rho_z^2} - \frac{4k_z}{k_z} \left( \frac{a_z}{\rho_z} \right)^2 \right) \right).
\end{align*}
\]

**Parameters characterizing the technological vibrations**

In order to achieve a fine grinding of the granular materials, the following technological parameters have to be considered and analyzed:

- the coefficient \( \Gamma \) of material throw-up, also called grinding mill characteristic parameter, is given by [6,11]:

\[
\Gamma = \frac{A \omega^2}{g}.
\]

This coefficient \( \Gamma \) takes values between 6...12;

- the frequency of technological vibrations must take values between (15...50) Hz, depending on the granular material and grinding fineness;

- the vertical amplitude of technological vibrations is about (3...10) mm.

An oscillating grinding mill operates properly if the vertical vibrations, called also technological vibrations, are predominant. Two case studies have to be considered:

- case study when the grinding mill is “centered”, i.e., the disturbing force center \( O \) coincides with mass center \( C \);
- case study when the grinding mill is “centered” and “well-balanced”, i.e., the horizontal plane of the mass center coincides with the plane of the disturbing force center and contains the vibration isolation elements.

For the centered grinding mill case study, the condition \( z_x = 0 \) yields the following relations for the amplitudes \( A_x \) and \( A_z \):

\[
\begin{align*}
A_x &= \frac{m \rho_x \omega^2}{4k_x} \frac{k_x}{k_z} \frac{a_z}{\rho_z}, \\
A_z &= \frac{m \rho_x \omega^2}{4k_z} \frac{k_z}{k_x} \frac{a_z}{\rho_z}.
\end{align*}
\]
The centered and balanced grinding mill case study is illustrated in figure 3, implying the conditions \( z_0 = 0 \) and \( a_y = 0 \), which involve \( A_x = 0 \) and the following relation for \( A_x \) amplitude:

\[
A_x = \frac{m_y r_0^2}{4k_z} \left( \frac{a_x}{\rho_y} \right)^2 - \left( \frac{\omega}{\omega_y} \right)^2 D,
\]

where

\[
D = \left( \frac{\omega}{\omega_y} \right)^4 \left[ k_z + k_y \left( \frac{a_x}{\rho_y} \right)^2 + \left( \frac{\omega}{\omega_y} \right)^2 k_x \right].
\]

In both cases, the amplitude \( A_z \) is given by relation (6).

**Parameters characterizing the vibration isolation subsystem**

The vibration isolation subsystem is composed of four rubber elements, having annular sections. These four rubber elements are mounted in parallel, working in compression in the vertical plane and working in shear in the horizontal plane (fig. 3).

The dynamic force amplitude \( F_{tx} \) on \( x \) direction, which is transmitted to the ground through the four elastic elements, is given by:

\[
F_{tx} = 4k_z \sqrt{A_x^2 + a_x^2 A_z^2}.
\]

The corresponding vibration isolation parameter is:

\[
I_x = 1 - \frac{F_{tx}}{P_0},
\]

it must take values between 0.70 \ldots 0.90.

The dynamic force amplitude \( F_{tz} \) on \( z \) direction, transmitted to the ground through the same four elastic elements, is:

\[
F_{tz} = 4k_z \sqrt{A_z^2 + a_z^2 A_x^2}.
\]

The corresponding vibration isolation parameter is:

\[
I_z = 1 - \frac{F_{tz}}{P_0},
\]

it must take values in the range 0.80 \ldots 0.95.

**Conclusions**

Based on the relations presented above and on the recommended values for the technological parameters, one can conclude that the motion of an uncentered and unbalanced grinding mill has three degrees of freedom, two of these degrees of freedom being coupled on \( x \) and \( \varphi \) directions; the best technical solution is the centered and balanced grinding mill (the one in fig. 3), described by the amplitude expressions (6) and (10); the vibration isolation parameters are given by (12) and (14).

The dynamic parameters, i.e., the maximum disturbing force \( P_0 \), the technological amplitude \( A_x \) and the technological angular frequency \( \omega_0 \), are determined by taking into account the grinding mill characteristic parameter \( \Gamma \) given by (7). Also, the grinding mill should operate in post-resonance, i.e., \( \omega = (3\ldots7) \rho \), where \( \rho \) is the natural angular frequency of the system corresponding to eigenmode \( x, z \) and \( \varphi \).

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