The Analysis of the Flexion Movement of the Horizontal Centrifuges Rotors with the Basket in Console

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The hereby paper presents an approach to the issue of the bending vibrations for the horizontal centrifuges with console mounted basket in the console. The shaft-basket system is designed as digital system with two degrees of freedom. It must be taken into account the influence of the gyroscopic moment.

Key words: centrifuge, vibrations, natural frequencies, critical speeds.

The centrifuge is a mechanical system composed of a flexible shaft with a negligible mass and a basket mounted on the shaft; the weight of the basket is much bigger than that of the shaft.

As regards the centrifuges, there are two issues:

a. the rigidity of the system, on which basis is determined the angular speed of the shaft, and its comparison with the critical angular speed that results from the determination of its own pulsation (this item was solved in [1]);

b. the flexion movement determined by placing the masses on the shaft (by placing the basket).

In some papers from technical literature [2, 3] the centrifuges are studied, but from the construction of basket point of view only. In these papers it is not approached the problem of the dynamic behavior of the centrifuges. In the paper [4] the rotors from the process equipment are studied, but from the stress calculation for shafts point of view only.

The hereby paper deals with the calculation of the critical angular speed (the critical speed) of the shaft. By knowing these critical speeds, it will be avoided the operation of the centrifuge at this speed or around this kind of speed.

Figure 1 presents the diagram of the horizontal centrifuge with the basket in console, where “G” represents the weight of the basket, together with the operating material (fluid).

As regards the study of the bending vibrations, the centrifuges may be considered in two ways:

- as a discrete system, with a finite number of degrees of freedom, which is composed of a flexible shaft of negligible weight, placed on two supports, and of a basket fixed on the shaft (between the supports or in the console);
- as a flexible shaft, considered a continuous medium (having an infinite number of degrees of freedom) placed on two supports and a basket fixed on the shaft (between the supports or in the console).

The paper uses the model with two degrees of freedom. This kind of model, although it represents a simplified kind of centrifuge, offers the possibility of a good and quick study of the dynamic behaviour of the centrifuge.

When the shaft is in horizontal position, with the basket fixed on the shaft, between the supports or in the console (figure 1), the weight of the basket shall not be taken into consideration. The weight force of the basket and, possibly, the moment of the gravitational force, trigger a static deformation of the shaft. The subsequent oscillatory movement is overlapping with this static deformation. Anyway, because the separation factor is \( F_c - G \geq 100 \) (the centrifugal force), it shall not be taken into account the weight of the basket when the rigidity of the shaft will be calculated.

Because there is an interest in the determination of the critical speeds, there will be studied only the undamped vibrations: the free and the forced vibrations. If the critical speeds cannot be avoided and the centrifuge is operating at almost critical speeds, the amplitudes of the bending vibrations can be reduced by installing a damper or a vibration dynamic absorber. The possibility to control the bending vibrations via these methods shall make the topic of a separate work.

The disturbing force may appear as a result of the fact that the weight centre of the basket is not on the rotation axis. This phenomenon can appear also during operation, because of the uneven material deposits on the basket or because of the damages during operation.

By choosing the system of axes presented in figure 2, with the \( Oy \) axis along the shaft, so in perpendicular position on the basket, the bending vibrations shall be studied in the \( xOy \) plane.

The centrifugal force that appears here is the following:

\[
F_c = m \cdot \Omega^2 \cdot OC = m \cdot \Omega^2 \cdot e
\]

where:
- \( m \) is the mass of the basket;
- \( \Omega \) - angular speed of the shaft-basket system;
- \( e \) - eccentricity \( OC = e \).

If relation (1) is projected on the axes (fig. 2,a) and if we take into account that \( \theta = \Omega t \), it shall result the following:

\[
F_c = \frac{m \cdot \Omega^2 \cdot OC}{\cos \theta}
\]

\[
F_c = \frac{m \cdot \Omega^2 \cdot OC}{\cos \theta} = \frac{m \cdot \Omega^2 \cdot e}{\cos \theta}
\]
The disturbing force is considered as follows:

\[
\begin{align*}
F_x &= F_c \cos \theta = m e \omega^2 \cos \Omega t \\
F_y &= F_c \sin \theta = m e \omega^2 \sin \Omega t \\
\end{align*}
\]  \hspace{1cm} (2)

If the basket is not fixed on the shaft, in the centre of mass, and when it takes place the reduction of the disturbance forces system in the fixing point \(O\) of the basket \(M_p = F_c h = me \omega^2 \cos \Omega t\), it will also appear a moment of the disturbing force.

In the case of small oscillations, the values of the \(\phi\) angle are small, so that the disturbing force \(F_p\) can be considered as perpendicular on the theoretical axis of the shaft \((F_c \cos \phi = F_x)\), while the axial component can be neglected \((F_c \sin \phi = 0; \ F_p \sin \phi \text{ is } 1000 \text{ times smaller than } F_c)\) (fig. 2, d).

Consequently, the elements of reduction of the system of disturbing forces in the \(O\) point are (fig. 2,e):

\[
\begin{align*}
F_{x} &= F_c \cos \theta = m e \omega^2 \cos \Omega t \\
F_{y} &= F_c \sin \theta = m e \omega^2 \sin \Omega t \\
M_{p} &= F_c h = me \omega^2 \cos \Omega t \\
\end{align*}
\]  \hspace{1cm} (3)

A factor that influences the values of the natural angular frequencies is the gyroscopic moment. When the basket is not fixed in the middle of the distance between the bearings or it is in console, the gyroscopic phenomenon will appear. The gyroscopic moment (fig. 3) modifies the bending deformation, so it modifies also the natural angular frequency and the critical speed.

The module of the gyroscopic moment has the following value [5]:

\[
M_g = (J_Y + J_z) \Omega^2 \phi
\]  \hspace{1cm} (4)

where:

- \(J_x = J_z\) represent the mass moments of inertia with respect to the \(Ox, Oz\) axes and respectively \(Oy\);
- \(\omega\) - the angular speed of the bending plane (the plane where the basket supports bends via vibrations);
- \(\Omega = k \ p\) - the angular speed of the shaft;
- \(k\) - the factor that induces the gyroscopic moment effect;
- \(\phi\) - the inclination of the shaft in the centre of mass section of the basket.

When the bending plane rotates in the same direction with the shaft is forward precession (fig. 3.a).

The console centrifugal basket

There are three situations in the case of the console centrifugal basket: the fixing point of the basket \(O\) stands between the centre of mass \((C)\) and the bearing (fig. 4,a); the centre of mass coincides with the fixing point of the basket (fig. 4,b); the centre of mass is between the fixing point of the basket and the bearing (fig. 4,c).
In order to write the differential equations of the movement, the method of the coefficients of influence is used. In this case, for the calculation of the R and M reactions in O (fig. 5,a), the basket shall be isolated and it shall be applied the d'Alembert's principle [6] (fig. 5,b). The following will be obtained:

\[
\begin{align*}
R &= mx\dot{y} \\
M &= mx\dot{y}(\cos \varphi + J_c \dot{\varphi}) \\
\end{align*}
\]  (5)

In order to shift from the \( x_c \) displacement of the centre of mass of the basket to the \( x \) displacement of the theoretical fixing point of the basket on the shaft, the following can be written (fig. 5,c):

\[
x_c = x + h \sin \varphi
\]  (6)

resulting the following, but taking into account that the \( \varphi \) angle has small values:

\[
\ddot{x_c} = \ddot{x} + h \ddot{\varphi}
\]  (7)

The equations of the R and M reactions, given by the equations (5), become as follows:

\[
\begin{align*}
R &= mx\ddot{y} + mh\ddot{\varphi} \\
M &= mh\ddot{y} + (mh^2 + J_c)\ddot{\varphi}
\end{align*}
\]  (8)

a. The case of the forward precession (fig. 5,d)

By using the method of the coefficients of influence, the following equations shall result:

\[
\begin{align*}
x &= \delta_x (R) + \delta_y (-M - M_x) \\
\dot{\varphi} &= \alpha_y (R) + \alpha_m (-M - M_x)
\end{align*}
\]  (9)

where:

\( \delta_x \) and \( \delta_y \) are the coefficients of influence that represent the displacements in the O point, generated by a force, and, respectively, by a moment equal with the unity;

\( \alpha_y, \alpha_m \) - the coefficients of influence that represent the rotations in the O point, these rotations being generated by a force, and, respectively, by a moment equal with the unity.

The equations related to the coefficients of influence can be found in [7]. By substituting \( R, M \) and \( M_g \) given in the previous equations and using the literal notations

\[
\begin{align*}
a_1 &= \delta_x + \delta_y h; \quad a_2 = mh^2 + J_c; \quad a_3 = J_c; \quad a_4 = \alpha_y + \alpha_m h
\end{align*}
\]

the following shall be obtained:

\[
\begin{align*}
x &= -a_1 m\ddot{x} - \delta_x mh + \delta_y a_1 \ddot{\varphi} - \delta_y a_3 \ddot{\varphi} \\
\dot{\varphi} &= -a_1 m\ddot{x} - \alpha_y mh + \alpha_m a_1 \ddot{\varphi} - \alpha_m a_3 \ddot{\varphi}
\end{align*}
\]  (10)

where:

\( \Omega \) represents the angular speed of the shaft;

\( J_c \) - the mechanic moment of inertia of the basket with respect to the \( Cz \) axis that passes through the centre of mass and that is perpendicular on the surface of the drawing;

\( J_y \) - the mechanical moment of inertia of the basket with respect to the \( Cy \) axis that passes through the centre of mass and which is the rotation theoretical axis.

Synchronous and in phase solutions like the following are searched:

\[
\begin{align*}
x &= X \cos(\omega t - \psi) \\
\dot{\varphi} &= \Phi \cos(\omega t - \psi)
\end{align*}
\]  (11)

By substituting these solutions in the equations (10), it shall be obtained the liniar, uniform, algebraic system with the unknown \( X \) and \( \Phi \):

\[
\begin{align*}
[a_1 m\omega^2 - J_x]X + [\delta_x mh + \delta_y a_1 \alpha_y - \delta_y a_3 \omega^2] \Phi &= 0 \\
[a_1 m\omega^2 X + \alpha_y mh + \alpha_m a_1 \alpha_y - \alpha_m a_3 \omega^2 - 1]\Phi &= 0
\end{align*}
\]  (12)

The determinant of the system must be 0 so that the system (12) to allow nonzero solutions. The equation of the natural angular frequencies is obtained from this condition:

\[
B_1 \omega^4 - B_2 \omega^2 + B_3 = 0
\]  (13)

where:

\[
\begin{align*}
B_1 &= a_1 m J_c \\
B_2 &= \delta_x m + a_1 mh + a_y a_2 + a_1 a_m \omega^2 \\
B_3 &= a_y a_3 \omega^2 + 1,
\end{align*}
\]  (14)

Where the following literal notations were used:

\[
\begin{align*}
a_i &= \delta_i a_y - \delta_y a_i; \quad a_y = \alpha_y + \delta_y
\end{align*}
\]

By solving this equation, it follows that:

\[
\begin{align*}
\omega_0 &= \sqrt{\frac{B_2 - \sqrt{B_2^2 - 4 B_1 B_3}}{2 B_1}} \\
\omega_2 &= \sqrt{\frac{B_2 + \sqrt{B_2^2 - 4 B_1 B_3}}{2 B_1}}
\end{align*}
\]  (15)  (16)

In the case of forced vibrations, it occur the elements of reduction of the system of the disturbing forces \( (F_p, M_p) \), encountered in the equations mentioned at item (3), figure 2e. Consequently, equations (9) are transformed into:

\[
\begin{align*}
x &= \delta_x (R + F_p) + \delta_y (-M - M_x + M_p) \\
\dot{\varphi} &= \alpha_y (R + F_p) + \alpha_m (-M - M_x + M_p)
\end{align*}
\]  (17)

By substituting \( R, M, F_p, M_p, M_g \) with the previously established expressions and by separating the terms, it shall be obtained the system of the non-homogeneous differential equations:

\[
\begin{align*}
-m\omega^2 a_y \cos \omega t &= -a_1 m\ddot{x} - a_y \ddot{\varphi} - x - \delta_y a_3 \omega^2 \varphi \\
-m\omega^2 a_y \cos \omega t &= -a_1 m\ddot{x} - a_y \ddot{\varphi} - 2a_3 \omega^2 \varphi
\end{align*}
\]  (18)
where the following literal notations were used:
\[ a_r = \delta_r mh + \delta_m \left( mh^2 + J_{cz} \right); \quad a_n = \alpha_r mh + \alpha_m \left( mh^2 + J_{cz} \right) \]

It is studied the permanent case of movement. There are proposed solutions like the following ones:

\[
\begin{align*}
    x &= X \cos \Omega t \\
    \varphi &= \Phi \cos \Omega t 
\end{align*}
\]

(19)

By substitution, it is obtained the non-homogeneous algebraic system:

\[
\begin{bmatrix}
    a_r \Omega^2 I - I & \Omega^2 [a_r - \delta_r a] \\
    a_n \Omega^2 X + \Omega [a_r - \alpha_r a] - \Omega^2 \left[ \alpha_m a_1 - ma_1 - a_n \right] + I
\end{bmatrix}
\begin{bmatrix}
    a_r \Omega^2 X
    \varphi
\end{bmatrix}
= -m \Omega^2 a_1
- m \Omega^2 [a_r - \alpha_r a] - \Omega^2 \left[ \alpha_m a_1 - ma_1 - a_n \right] + I
\]

(20)

which, if solved, leads to the expressions afferent to the amplitude of the movements, in the case of forced vibrations:

\[ X = \frac{m \Omega^2 a_1 + m \Omega^2 a_1 [a_r - J_{cz}]}{-m \Omega^2 a_1 [a_r - J_{cz}] + \Omega^2 [\alpha_m a_1 - ma_1 - a_n] + I} \]

\[ \varphi = \frac{m \Omega^2 a_1}{-m \Omega^2 a_1 [a_r - J_{cz}] + \Omega^2 [\alpha_m a_1 - ma_1 - a_n] + I} \]

(22)

\[ \varphi = -m \Omega^2 a_1 [a_r - J_{cz}] + \Omega^2 [\alpha_m a_1 - ma_1 - a_n] + I \]

(21)

\[ \varphi = -m \Omega^2 a_1 \]

b. The case of the backward precession

By following the same calculation steps as in the case of the forward precession, there will be obtained the natural angular frequencies and the amplitudes of the permanent movement (in the case of forced vibrations). The only difference is that the expression of the gyroscopic moment is substituted in all the equations: \( M = [J_0 - J_1] \Omega^2 \varphi \)

Substituted by \( M = -[J_0 + J_1] \Omega^2 \varphi \). In other words, the expression \( a_r = J_0 - J_{cz} \) shall be substituted by \( a_r = -[J_0 + J_{cz}] \) in all equations.

The fixing point of the basket on the shaft coincides with the centre of mass

This case is characterized by the fact that \( h = 0 \). Consequently, the equations by which are determined the natural angular frequencies and the amplitudes of the permanent movement (in the case of forced vibrations), regarding both the forward and the backward precession, are obtained from the relations (15), (16), (21), (22) by substituting \( h \) with \( h = 0 \).

The centre of mass is between the fixing point of the basket on the shaft and bearing

In this case, the study of the vibrations is similar with the one mentioned at item 2.1. The difference is that \( h < 0 \). Consequently, the equations by which are determined the natural angular frequencies and the amplitudes of permanent movement (in the case of forced vibrations), regarding both the forward and backward precession, are obtained from the relations (15), (16), (21), (22) by substituting \( h \) with \( -h \).

Example

It is considered a horizontal console centrifugal basket. The basket has the shape of a cylinder. The geometrical and mechanical characteristics of the shaft-basket system are the following:

- length of the shaft: 0.8 m;
- diameter of the shaft: 0.08 m;
- diameter of the basket: 0.5 m

- length of the basket: 0.12 m;
- distance from the bearing to the fixing point of the basket: 0.05 m;
- density of the material of the shaft and basket: 7800 Kg/m³;
- longitudinal modulus of elasticity (Young’s modulus): 21 x 10¹⁰ N/m²;
- distance from the longitudinal axis of the basket to the centre of mass (\( e \)): 0.003 m.

As regards the coefficients of influence, there were obtained the following values:

\[ \delta_r = 1.3877 \times 10^{-6} \text{ m·N}^{-1}; \delta_n = -2.5904 \times 10^{-8} \text{ N}^{-1}; \]

\[ \alpha_r = -2.5904 \times 10^{-8} \text{ N}^{-1}; \alpha_n = 5.2055 \times 10^{-7} \text{ m·N}^{-1}. \]

The mechanical moments of inertia:

\[ J_{cy} = 1.8755 \text{ kg·m}²; J_{cz} = 1.0628 \text{ kg·m}². \]

It is considered the case when the fixing point of the basket is between the centre of mass and the bearing. On the basis of the relations (15), (16) the natural angular frequencies were determined. Because the second angular frequency has a higher value, the first angular frequency, meaning the fundamental natural angular frequency (\( \omega_1 \)), shall be taken into consideration. Figure 6 presents the variation of the fundamental natural frequency, \( \omega_1 = \frac{\omega}{2\pi} \), depending on the speed. In order to determine the critical speeds, there have been presented the first four harmonics of the disturbing force. It can be observed that, regarding the considered operational domain of the centrifuge (1200 ÷ 2800 r.p.m.), there are occurring critical speeds which correspond to the third and fourth harmonics. At the same time, it also can be observed that the forward precession leads to the increasing of the critical speeds, while the backward precession triggers the decreasing of the critical speeds.

Figure 7 envisages the variation of the fundamental natural frequency, depending on the distance from the centre of mass to the fixing point of the basket, at 2000 r.p.m. The variation range is considered between -0.06 ÷ 0.06 m. In this way, there are covered also the cases mentioned at items 2.2. and 2.3. It was considered \( h < 0 \) when the fixing point of the basket stands between the centre of mass and the bearing. It can be observed that the lowest natural frequency values are obtained when \( h < 0 \), in other words, when the fixing point of the basket on the shaft is situated between the centre of mass and
Fig. 7. The variation of the fundamental natural frequency ($\omega_1$) depending on the position of the fixing point of the basket on the shaft, in comparison with the centre of mass.

Fig. 8. The variation of the fundamental natural frequency ($\omega_1$) depending on the lateral wall thickness of the basket.

Fig. 9. The variation of the fundamental natural frequency ($\omega_1$) depending on the speed and on the position of the fixing point of the basket, in comparison with the centre of mass.

Fig. 10. The variation of the fundamental natural frequency ($\omega_1$) depending on the speed and on the lateral wall thickness of the basket.

bearing. When the distance from the centre of mass to the fixing point of the basket on the shaft is decreasing and the fixing point of the basket passes after the centre of mass ($h>0$) the fundamental natural frequency is increasing. In the case of the forward precession, the increase is of 16%, while in the case of the backward precession, the increase is of 15%.

While the centrifuge is in operation, the mass and the mechanical inertia moments of the basket will be modified as a result of the deposits on the side walls of the basket.

Figure 8 represents the variation of the fundamental natural frequency, depending on the lateral wall thickness of the basket, for a speed of 2000 r.p.m. The variation range is between 0.01–0.03 m. It can be observed that the thicker the lateral wall of the basket, the more decreases the fundamental natural frequency. This decrease is of approximately 29%, in the case of backward precession, and of approximately 24%, in the case of forward precession.

Figure 9 envisages the variation of the fundamental natural frequency, depending on the speed and on the distance from the centre of mass to the fixing point of the basket, for a 0.01 m thickness of the basket wall. It can be noticed a significant decrease of the natural frequency, corresponding to the backward precession, when there are high speeds and negative values of the distance from the centre of mass to the fixing point of the basket. If it is designed a plane adequate to one of the $h$ values, it shall be obtained a diagram similar to the one presented in figure 6.

Figure 10 represents the variation of the fundamental natural frequency, depending on the speed and on the wall thickness of the basket. Taking into account that, in the case of backward precession, as the speed is increasing the natural frequency is decreasing, it may be possible that the critical speeds to be highly reduced, if the wall of the basket becomes thicker (as a result of the deposits), triggering the possible critical speeds in the domain of adequate operation of some inferior harmonics (for instance, the second harmonic).

**Conclusions**

Due to the constructive and operational features, during the operation of the centrifuges, it may appear modifications of the mechanical characteristics of these centrifuges, such as the modification of the position of the centers of mass and of the weight of the basket. The uneven deposits represent the cause. The result of these deposits is the alteration of the dynamic behaviour of the centrifuge which consists in vibrations with amplitudes that cannot be neglected and that involve unusual sounds, wears in the bearings and a decreased quality of the products. The fact that the experts take into account a data set during designing and that this data set modifies during operation, constitutes an issue that cannot allow the predictions of the dynamic behaviour of the centrifuge.
The hereby document submits a model for the study of the bending vibrations of the centrifuges, in general, and of the console centrifugal baskets, in particular. Although it is a simplified model, it gives the opportunity to the experts to understand the occurring phenomena and to take measures against such phenomena, or, if possible, to avoid them during operation.

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